# Joins → Aggregates → Optimization

https://fdbresearch.github.io

**Dan Olteanu**

PhD Open School
University of Warsaw
November 22, 2018

# Acknowledgements

Some work reported in this course has been done in the context of the FDB project, LogicBlox, and RelationalAI by

- Zavodný, Schleich, Kara, Nikolic, Zhang, Ciucanu, and Olteanu (Oxford)
- Abo Khamis and Ngo (RelationalAI), Nguyen (U. Michigan)

Some of the following slides are derived from presentations by

- Abo Khamis (optimization diagrams)
- Kara (covers, $\text{IVM}^{\epsilon}$, and many graphics)
- Ngo (functional aggregate queries)
- Schleich (performance and quizzes)

Lastly, Kara and Schleich proofread the slides.

I would like to thank them for their support!

# Goal of This Course

Introduction to a principled approach to in-database computation

This course starts where mainstream databases courses finish.

- **Part 1: Joins**
  - Basic building blocks in query languages. Studied extensively.
  - Systematic study of redundancy in the computation and representation of join results [OZ12,OZ15,KO18]
  - Worst-case optimal join algorithms [NPRR12,NRR13,V14,OZ15,ANS17]
- Part 2: Aggregates
- Part 3: Optimization

# Outline of Part 1: Joins

- Introduction by Examples
- Decompositions and Variable Orders
- Size Bounds for Join Results
- Worst-Case Optimal Join Algorithms
- Further Work and References
- Quiz

# Join Queries

$$\underbrace{Q(\boldsymbol{A}_1 \cup \cdots \cup \boldsymbol{A}_n)}_{\text{head}} = \underbrace{R_1(\boldsymbol{A}_1), \ldots, R_n(\boldsymbol{A}_n)}_{\text{body}}$$

- Query variables: $\boldsymbol{A}_1 \cup \cdots \cup \boldsymbol{A}_n$. *All variables in the body occur in the head.*
- Relational atoms: $R_1, \ldots, R_n$
- Natural join: Same variable occurs in different relational atoms

Examples of bodies of queries used in the following slides:

- Path: $O(customer, day, dish), D(dish, item), I(item, price)$
- Path: $R_1(A, B), R_2(B, C), R_3(C, D)$
- Acyclic: $R(A, B, C), S(A, B, D), T(A, E), U(E, F)$.
- Triangle: $R_1(A, B), R_2(A, C), R_3(B, C)$
- Loop: $R(A_1, A_2), S(A_2, A_3), T(A_3, A_4), W(A_4, A_1)$

# Join Example: Itemized Customer Orders

Orders (O for short)

| customer | day | dish |
|---|---|---|
| Elise | Monday | burger |
| Elise | Friday | burger |
| Steve | Friday | hotdog |
| Joe | Friday | hotdog |

Dish (D for short)

| dish | item |
|---|---|
| burger | patty |
| burger | onion |
| burger | bun |
| hotdog | bun |
| hotdog | onion |
| hotdog | sausage |

Items (I for short)

| item | price |
|---|---|
| patty | 6 |
| onion | 2 |
| bun | 2 |
| sausage | 4 |

Consider the natural join of the above relations:

O(customer, day, dish), D(dish, item), I(item, price)

| customer | day | dish | item | price |
|---|---|---|---|---|
| Elise | Monday | burger | patty | 6 |
| Elise | Monday | burger | onion | 2 |
| Elise | Monday | burger | bun | 2 |
| Elise | Friday | burger | patty | 6 |
| Elise | Friday | burger | onion | 2 |
| Elise | Friday | burger | bun | 2 |
| ... | ... | ... | ... | ... |

# Join Example: Listing the Triangles in the Database

$R_1$

| $A$ | $B$ |
|---|---|
| $a_0$ | $b_0$ |
| $a_0$ | $\ldots$ |
| $a_0$ | $b_m$ |
| $a_1$ | $b_0$ |
| $\ldots$ | $b_0$ |
| $a_m$ | $b_0$ |

$R_2$

| $A$ | $C$ |
|---|---|
| $a_0$ | $c_0$ |
| $a_0$ | $\ldots$ |
| $a_0$ | $c_m$ |
| $a_1$ | $c_0$ |
| $\ldots$ | $c_0$ |
| $a_m$ | $c_0$ |

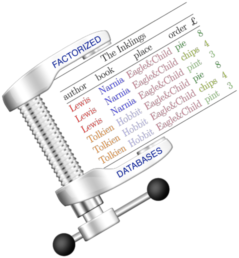
$R_3$

| $B$ | $C$ |
|---|---|
| $b_0$ | $c_0$ |
| $b_0$ | $\ldots$ |
| $b_0$ | $c_m$ |
| $b_1$ | $c_0$ |
| $\ldots$ | $c_0$ |
| $b_m$ | $c_0$ |

$R_1(A, B), R_2(A, C), R_3(B, C)$

| $A$ | $B$ | $C$ |
|---|---|---|
| $a_0$ | $b_0$ | $c_0$ |
| $a_0$ | $b_0$ | $\ldots$ |
| $a_0$ | $b_0$ | $c_m$ |
| $a_0$ | $b_1$ | $c_0$ |
| $a_0$ | $\ldots$ | $c_0$ |
| $a_0$ | $b_m$ | $c_0$ |
| $a_1$ | $b_0$ | $c_0$ |
| $\ldots$ | $b_0$ | $c_0$ |
| $a_1$ | $b_0$ | $c_0$ |

# Outline of Part 1: Joins

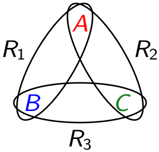


# Join Hypergraphs

We associate a (multi)hypergraph $\mathcal{H} = (\mathcal{V}, \mathcal{E})$ with every join query $Q$

- Each variable in $Q$ is a node in $\mathcal{V}$
- The set of variables of each relation symbol in $Q$ is a (hyper)edge in $\mathcal{E}$

Example: Triangle query $R_1(A, B), R_2(A, C), R_3(B, C)$



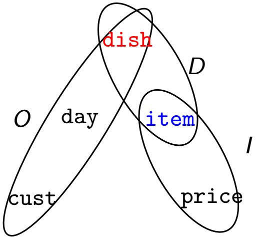

- $\mathcal{V} = \{A, B, C\}$
- $\mathcal{E} = \{\{A, B\}, \{A, C\}, \{B, C\}\}$

# Join Hypergraphs

We associate a (multi)hypergraph $\mathcal{H} = (\mathcal{V}, \mathcal{E})$ with every join query $Q$

- Each variable in $Q$ is a node in $\mathcal{V}$
- The set of variables of each relation symbol in $Q$ is a (hyper)edge in $\mathcal{E}$

Example: Order query $O(\texttt{cust}, \texttt{day}, \texttt{dish}), D(\texttt{dish}, \texttt{item}), I(\texttt{item}, \texttt{price})$




- $\mathcal{V} = \{\texttt{cust}, \texttt{day}, \texttt{dish}, \texttt{item}, \texttt{price}\}$
- $\mathcal{E} = \{\{\texttt{cust}, \texttt{day}, \texttt{dish}\}, \{\texttt{dish}, \texttt{item}\}, \{\texttt{item}, \texttt{price}\}\}$

# Hypertree Decompositions

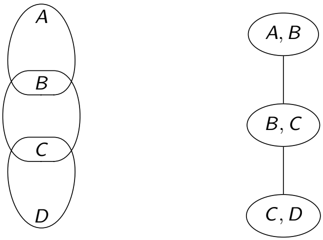
**Definition**[GLS99]: A (hypertree) decomposition $\mathcal{T}$ of the hypergraph $(\mathcal{V}, \mathcal{E})$ of a query $Q$ is a pair $(T, \chi)$, where

- $T$ is a tree
- $\chi$ is a function mapping each node in $T$ to a subset of $\mathcal{V}$ called *bag*.

Properties of a decomposition $\mathcal{T}$:

- *Coverage*: $\forall e \in \mathcal{E}$, there must be a node $t \in T$ such that $e \subseteq \chi(t)$.
- *Connectivity*: $\forall v \in \mathcal{V}$, $\{t \mid t \in T, v \in \chi(t)\}$ forms a connected subtree.

The hypergraph of the query $R_1(A, B), R_2(B, C), R_3(C, D)$ — A hypertree decomposition



# Hypertree Decompositions

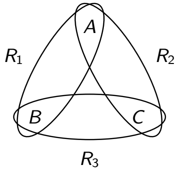
**Definition**[GLS99]: A (hypertree) decomposition $\mathcal{T}$ of the hypergraph $(\mathcal{V}, \mathcal{E})$ of a query $Q$ is a pair $(T, \chi)$, where

- $T$ is a tree
- $\chi$ is a function mapping each node in $T$ to a subset of $\mathcal{V}$ called *bag*.

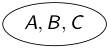
Properties of the decomposition $\mathcal{T}$:

- *Coverage*: $\forall e \in \mathcal{E}$, there must be a node $t \in T$ such that $e \subseteq \chi(t)$.
- *Connectivity*: $\forall v \in \mathcal{V}$, $\{t \mid t \in T, v \in \chi(t)\}$ forms a connected subtree.

The hypergraph of the triangle query $R_1(A, B), R_2(A, C), R_3(B, C)$




A hypertree decomposition



# Variable Orders

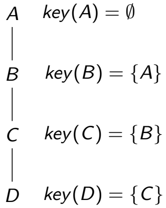
**Definition**[OZ15]: A *variable order* $\Delta$ for a query $Q$ is a pair $(F, key)$, where

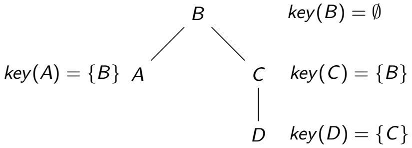
- $F$ is a rooted forest with one node per variable in $Q$
- $key$ is a function mapping each variable $A$ to a subset of its ancestor variables in $F$.

Properties of a variable order $\Delta$ for $Q$:

- For each relation symbol, its variables lie along the same root-to-leaf path in $F$. For any such variables $A$ and $B$, $A \in key(B)$ if $A$ is an ancestor of $B$.
- For every child $B$ of $A$, $key(B) \subseteq key(A) \cup \{A\}$.

Possible variable orders for the path query $R_1(A, B), R_2(B, C), R_3(C, D)$:



# Variable Orders

**Definition**[OZ15]: A *variable order* $\Delta$ for a query $Q$ is a pair $(F, key)$, where

- $F$ is a rooted forest with one node per variable in $Q$
- $key$ is a function mapping each variable $A$ to a subset of its ancestor variables in $F$.

Properties of a variable order $\Delta$ for $Q$:

- For each relation symbol, its variables lie along the same root-to-leaf path in $F$. For any such variables $A$ and $B$, $A \in key(B)$ if $A$ is an ancestor of $B$.
- For every child $B$ of $A$, $key(B) \subseteq key(A) \cup \{A\}$.

Possible variable orders for the triangle query $R_1(A, B), R_2(A, C), R_3(B, C)$:

$A$ $key(A) = \emptyset$
|
$B$ $key(B) = \{A\}$
|
$C$ $key(C) = \{A, B\}$

$B$ $key(B) = \emptyset$
|
$A$ $key(A) = \{B\}$
|
$C$ $key(C) = \{A, B\}$

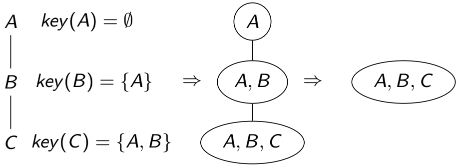
$C$ $key(C) = \emptyset$
|
$B$ $key(B) = \{C\}$
|
$A$ $key(A) = \{B, C\}$

# Hypertree Decompositions $\Leftrightarrow$ Variable Orders

From variable order $\Delta$ to hypertree decomposition $\mathcal{T}$: [OZ15]

- For each node $A$ in $\Delta$, create a bag $key(A) \cup \{A\}$.
- The bag for $A$ is connected to the bags for its children and parent.
- Optionally, remove redundant bags

Example: Triangle query $R_1(A, B), R_2(A, C), R_3(B, C)$

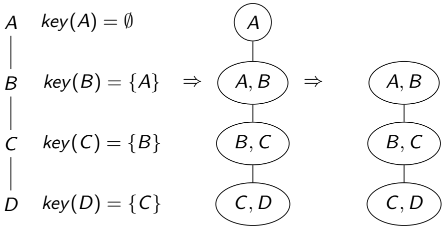


# Hypertree Decompositions $\Leftrightarrow$ Variable Orders

From variable order $\Delta$ to hypertree decomposition $\mathcal{T}$: [OZ15]

- For each node $A$ in $\Delta$, create a bag $key(A) \cup \{A\}$.
- The bag for $A$ is connected to the bags for its children and parent.
- Optionally, remove redundant bags

Example: Path query $R_1(A, B), R_2(B, C), R_3(C, D)$

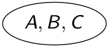


# Hypertree Decompositions ⇔ Variable Orders

From hypertree decomposition $\mathcal{T}$ to variable order $\Delta$: [OZ15]

- Create a node $A$ in $\Delta$ for a variable $A$ in the top bag in $\mathcal{T}$
- Recurse with $\mathcal{T}$ where $A$ is removed from all bags in $\mathcal{T}$.
- If top bag empty, then recurse independently on each of its child bags and create children of $A$ in $\Delta$
- Update *key* for each variable at each step.

Example: Triangle query $R_1(A, B), R_2(A, C), R_3(B, C)$



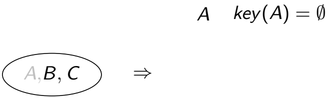
# Hypertree Decompositions ⇔ Variable Orders

From hypertree decomposition $\mathcal{T}$ to variable order $\Delta$: [OZ15]

- Create a node $A$ in $\Delta$ for a variable $A$ in the top bag in $\mathcal{T}$
- Recurse with $\mathcal{T}$ where $A$ is removed from all bags in $\mathcal{T}$.
- If top bag empty, then recurse independently on each of its child bags and create children of $A$ in $\Delta$
- Update *key* for each variable at each step.

Example: Triangle query $R_1(A, B), R_2(A, C), R_3(B, C)$

*Step 1:*
*$A$ is removed from $\mathcal{T}$*
*and inserted into $\Delta$*



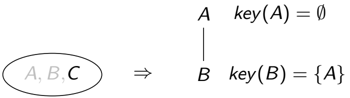
# Hypertree Decompositions ⇔ Variable Orders

From hypertree decomposition $\mathcal{T}$ to variable order $\Delta$: [OZ15]

- Create a node $A$ in $\Delta$ for a variable $A$ in the top bag in $\mathcal{T}$
- Recurse with $\mathcal{T}$ where $A$ is removed from all bags in $\mathcal{T}$.
- If top bag empty, then recurse independently on each of its child bags and create children of $A$ in $\Delta$
- Update *key* for each variable at each step.

Example: Triangle query $R_1(A, B), R_2(A, C), R_3(B, C)$

*Step 2:*
*$B$ is removed from $\mathcal{T}$*
*and inserted into $\Delta$*



# Hypertree Decompositions ⇔ Variable Orders

From hypertree decomposition $\mathcal{T}$ to variable order $\Delta$: [OZ15]

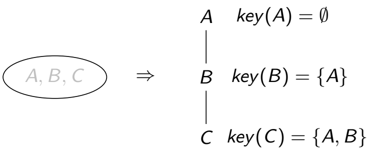
- Create a node $A$ in $\Delta$ for a variable $A$ in the top bag in $\mathcal{T}$
- Recurse with $\mathcal{T}$ where $A$ is removed from all bags in $\mathcal{T}$.
- If top bag empty, then recurse independently on each of its child bags and create children of $A$ in $\Delta$
- Update *key* for each variable at each step.

Example: Triangle query $R_1(A, B), R_2(A, C), R_3(B, C)$

*Step 3:*

*$C$ is removed from $\mathcal{T}$*

*and inserted into $\Delta$*

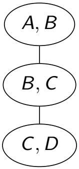


# Hypertree Decompositions ⇔ Variable Orders

From hypertree decomposition $\mathcal{T}$ to variable order $\Delta$: [OZ15]

- Create a node $A$ in $\Delta$ for a variable $A$ in the top bag in $\mathcal{T}$
- Recurse with $\mathcal{T}$ where $A$ is removed from all bags in $\mathcal{T}$.
- If top bag empty, then recurse independently on each of its child bags and create children of $A$ in $\Delta$
- Update *key* for each variable at each step.

Example: Path query $R_1(A, B), R_2(B, C), R_3(C, D)$



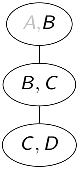
# Hypertree Decompositions $\Leftrightarrow$ Variable Orders

From hypertree decomposition $\mathcal{T}$ to variable order $\Delta$: [OZ15]

- Create a node $A$ in $\Delta$ for a variable $A$ in the top bag in $\mathcal{T}$
- Recurse with $\mathcal{T}$ where $A$ is removed from all bags in $\mathcal{T}$.
- If top bag empty, then recurse independently on each of its child bags and create children of $A$ in $\Delta$
- Update *key* for each variable at each step.

Example: Path query $R_1(A, B), R_2(B, C), R_3(C, D)$

*Step 1:*
*$A$ is removed from $\mathcal{T}$*
*and inserted into $\Delta$*




$\Rightarrow$

$A \quad key(A) = \emptyset$

# Hypertree Decompositions ⇔ Variable Orders

From hypertree decomposition $\mathcal{T}$ to variable order $\Delta$: [OZ15]

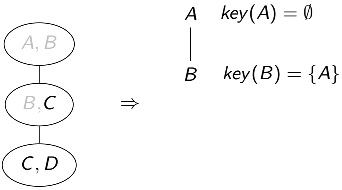
- Create a node $A$ in $\Delta$ for a variable $A$ in the top bag in $\mathcal{T}$
- Recurse with $\mathcal{T}$ where $A$ is removed from all bags in $\mathcal{T}$.
- If top bag empty, then recurse independently on each of its child bags and create children of $A$ in $\Delta$
- Update *key* for each variable at each step.

Example: Path query $R_1(A,B), R_2(B,C), R_3(C,D)$

*Step 2:*

*$B$ is removed from $\mathcal{T}$*

*and inserted into $\Delta$*



# Hypertree Decompositions ⇔ Variable Orders

From hypertree decomposition $\mathcal{T}$ to variable order $\Delta$: [OZ15]

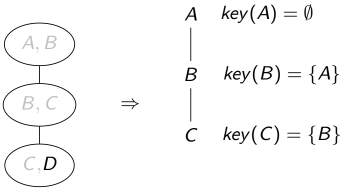
- Create a node $A$ in $\Delta$ for a variable $A$ in the top bag in $\mathcal{T}$
- Recurse with $\mathcal{T}$ where $A$ is removed from all bags in $\mathcal{T}$.
- If top bag empty, then recurse independently on each of its child bags and create children of $A$ in $\Delta$
- Update *key* for each variable at each step.

Example: Path query $R_1(A, B), R_2(B, C), R_3(C, D)$

*Step 3:*

*$C$ is removed from $\mathcal{T}$*

*and inserted into $\Delta$*

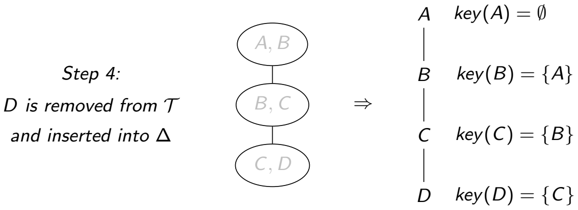


# Hypertree Decompositions ⇔ Variable Orders

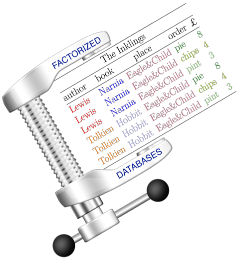
From hypertree decomposition $\mathcal{T}$ to variable order $\Delta$: [OZ15]

- Create a node $A$ in $\Delta$ for a variable $A$ in the top bag in $\mathcal{T}$
- Recurse with $\mathcal{T}$ where $A$ is removed from all bags in $\mathcal{T}$.
- If top bag empty, then recurse independently on each of its child bags and create children of $A$ in $\Delta$
- Update *key* for each variable at each step.

Example: Path query $R_1(A,B), R_2(B,C), R_3(C,D)$



# Outline of Part 1: Joins



# How Can We Bound the Size of the Join Result?

Example: the path query $R_1(A, B), R_2(B, C), R_3(C, D)$

- Assumption: All relations have size $N$.

- The query result is included in the result of $R_1(A, B), R_3(C, D)$
  - Its size is upper bounded by $N^2 = |R_1| \times |R_3|$
  - All variables are "covered" by the relations $R_1$ and $R_3$

- There are databases for which the result size is at least $N^2$
  - Let $R_1 = [N] \times \{1\}, R_2 = \{1\} \times [N], R_3 = [N] \times \{1\}$.

# How Can We Bound the Size of the Join Result?

Example: the path query $R_1(A, B), R_2(B, C), R_3(C, D)$

- Assumption: All relations have size $N$.

- The query result is included in the result of $R_1(A, B), R_3(C, D)$
  - Its size is upper bounded by $N^2 = |R_1| \times |R_3|$
  - All variables are "covered" by the relations $R_1$ and $R_3$

- There are databases for which the result size is at least $N^2$
  - Let $R_1 = [N] \times \{1\}, R_2 = \{1\} \times [N], R_3 = [N] \times \{1\}$.

- Conclusion: Size of the query result is $\Theta(N^2)$ for some input classes

# How Can We Bound the Size of the Join Result?

Example: the triangle query $R_1(A, B), R_2(A, C), R_3(B, C)$

- Assumption: All relations have size $N$.

- The query result is included in the result of $R_1(A, B), R_3(B, C)$
  - Its size is upper bounded by $N^2 = |R_1| \times |R_3|$
  - All variables are "covered" by the relations $R_1$ and $R_3$

- There are databases for which the result size is at least $N$
  - Let $R_1 = [N] \times \{1\}, R_2 = [N] \times \{1\}, R_3 \supseteq \{(1, 1)\}$

# How Can We Bound the Size of the Join Result?

Example: the triangle query $R_1(A, B), R_2(A, C), R_3(B, C)$

- Assumption: All relations have size $N$.

- The query result is included in the result of $R_1(A, B), R_3(B, C)$
  - Its size is upper bounded by $N^2 = |R_1| \times |R_3|$
  - All variables are "covered" by the relations $R_1$ and $R_3$

- There are databases for which the result size is at least $N$
  - Let $R_1 = [N] \times \{1\}, R_2 = [N] \times \{1\}, R_3 \supseteq \{(1, 1)\}$

- Conclusion: Size gap between the $N^2$ upper bound and the $N$ lower bound!

**Question: Can we close this gap and give tight size bounds?**

# Edge Covers and Independent Sets

We can generalize the previous examples as follows:

For the size upper bound:

- Cover all nodes (variables) by $k$ edges (relations) $\Rightarrow$ size $\leq N^k$.
- This is an edge cover of the query hypergraph!

For the size lower bound:

- $m$ independent nodes $\Rightarrow$ construct database such that size $\geq N^m$.
- This is an independent set of the query hypergraph!

$$\max_m = |\mathrm{IndependentSet}(Q)| \leq |\mathrm{EdgeCover}(Q)| = \min_k$$

$\max_m$ and $\min_k$ do not necessarily meet!

Can we further refine this analysis?

# The Fractional Edge Cover Number $\rho^*(Q)$

The two bounds meet if we take their fractional versions [AGM08]

- *Fractional* edge cover of $Q$ with weight $k \Rightarrow$ size $\leq N^k$.
- *Fractional* independent set with weight $m \Rightarrow$ size $\geq N^m$.

By duality of linear programming:

$$\max_m = |\mathrm{FractionalIndependentSet}(Q)| = |\mathrm{FractionalEdgeCover}(Q)| = \min_k$$

- This is the fractional edge cover number $\rho^*(Q)$!

**For query $Q$ and database of size $N$, the query result has size $O(N^{\rho^*(Q)})$.**

# The Fractional Edge Cover Number $\rho^*(Q)$

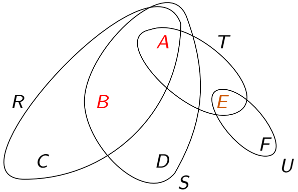
For a join query $Q(\boldsymbol{A}_1 \cup \cdots \cup \boldsymbol{A}_n) = R_1(\boldsymbol{A}_1), \ldots, R_n(\boldsymbol{A}_n)$,
$\rho^*(Q)$ is the cost of an optimal solution to the linear program:

$$
\begin{aligned}
\text{minimize} \quad & \textstyle\sum_{i \in [n]} x_{R_i} \\
\text{subject to} \quad & \textstyle\sum_{i:\text{edge } R_i \text{ covers node } A} x_{R_i} \geq 1 \quad \forall A \in \bigcup_{j \in [n]} \boldsymbol{A}_j, \\
& x_{R_i} \geq 0 \qquad\qquad\qquad \forall i \in [n].
\end{aligned}
$$

- $x_{R_i}$ is the weight of edge (relation) $R_i$ in the hypergraph of $Q$
- Each node (variable) has to be covered by edges with sum of weights $\geq 1$
- In the integer program variant for the edge cover, $x_{R_i} \in \{0, 1\}$

# Example: Compute the Fractional Edge Cover (1/3)

Consider the join query $Q$: $R(A, B, C), S(A, B, D), T(A, E), U(E, F)$.



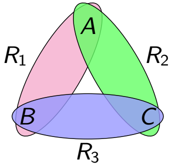

- The three edges $R, S, U$ can cover all nodes.
  $\mathrm{FractionalEdgeCover}(Q) \leq 3$
- Each node $C$, $D$, and $F$ must be covered by a distinct edge.
  $\mathrm{FractionalIndependentSet}(Q) \geq 3$

$\Rightarrow \rho^*(Q) = 3$

$\Rightarrow$ Size $\leq N^3$ and for some inputs is $\Theta(N^3)$.

# Example: Compute the Fractional Edge Cover (2/3)

Consider the triangle query: $R_1(A, B), R_2(A, C), R_3(B, C)$.




$$
\begin{array}{lccccccl}
\textit{minimize} & x_{R_1} + x_{R_2} + x_{R_3} \\
\textit{subject to} \\
A: & x_{R_1} & + & x_{R_2} & & & \geq 1 \\
B: & x_{R_1} & & & + & x_{R_3} & \geq 1 \\
C: & & & x_{R_2} & + & x_{R_3} & \geq 1 \\
 & x_{R_1} \geq 0 & & x_{R_2} \geq 0 & & x_{R_3} \geq 0
\end{array}
$$

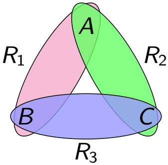
Our previous size upper bound was $N^2$:

- This is obtained by setting any two of $x_{R_1}, x_{R_2}, x_{R_3}$ to 1.

What is the fractional edge cover number for the triangle query?

# Example: Compute the Fractional Edge Cover (2/3)

Consider the triangle query: $R_1(A, B), R_2(A, C), R_3(B, C)$.




$$
\begin{array}{lccccccc}
\textit{minimize} & x_{R_1} + x_{R_2} + x_{R_3} \\
\textit{subject to} \\
A: & x_{R_1} & + & x_{R_2} & & & \geq 1 \\
B: & x_{R_1} & & & + & x_{R_3} & \geq 1 \\
C: & & & x_{R_2} & + & x_{R_3} & \geq 1 \\
 & x_{R_1} \geq 0 & & x_{R_2} \geq 0 & & x_{R_3} \geq 0 &
\end{array}
$$

Our previous size upper bound was $N^2$:

- This is obtained by setting any two of $x_{R_1}, x_{R_2}, x_{R_3}$ to 1.

What is the fractional edge cover number for the triangle query?

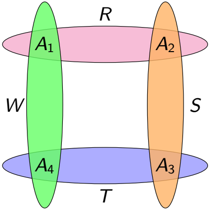
We can do better: $x_{R_1} = x_{R_2} = x_{R_3} = 1/2$. Then, $\rho^* = 3/2$.

Lower bound reaches $N^{3/2}$ for $R_1 = R_2 = R_3 = [\sqrt{N}] \times [\sqrt{N}]$.

# Example: Compute the Fractional Edge Cover (3/3)

Consider the (4-cycle) join: $R(A_1, A_2), S(A_2, A_3), T(A_3, A_4), W(A_4, A_1)$.

The linear program for its fractional edge cover number:




$$
\begin{array}{lccccccc}
\textit{minimize } x_R + x_S + x_T + x_W \\
\textit{subject to} \\
A_1: & x_R & & & & + & x_W & \geq 1 \\
A_2: & x_R & + & x_S & & & & \geq 1 \\
A_3: & & & x_S & + & x_T & & \geq 1 \\
A_4: & & & & & x_T & + \; x_W & \geq 1 \\
& x_R \geq 0 & & x_S \geq 0 & & x_T \geq 0 & x_W \geq 0 &
\end{array}
$$

Possible solution: $x_R = x_T = 1$. Another solution: $x_S = x_W = 1$. Then, $\rho^* = 2$.

Lower bound reaches $N^2$ for $R = T = [N] \times \{1\}$ and $S = W = \{1\} \times [N]$.

# Historical Note on the Fractional Edge Cover Number

Tight size bounds via $\rho*$ have been known from earlier works in other contexts:

- (special case) Loomis-Whitney inequality [LW49]
- (general case) number of occurrences of a subgraph in a graph [A81]
- generalization of Loomis-Whitney that subsumes the AGM bound [BT95]

Recent insightful travel through the history of this result [H18]

# Refinement under Cardinality Constraints

Common case in practice:

- Relations have different sizes
- Small-size projections of relations may be added to the join query

Recall the linear program for computing the fractional edge cover number $\rho^*(Q)$ of a join query $Q(\boldsymbol{A}_1 \cup \cdots \cup \boldsymbol{A}_n) = R_1(\boldsymbol{A}_1), \ldots, R_n(\boldsymbol{A}_n)$:

$$
\begin{aligned}
\text{minimize} \quad & \textstyle\sum_{i \in [n]} x_{R_i} \\
\text{subject to} \quad & \textstyle\sum_{i:\text{edge } R_i \text{ covers node } A} x_{R_i} \geq 1 \quad \forall A \in \bigcup_{j \in [n]} \boldsymbol{A}_j, \\
& x_{R_i} \geq 0 \qquad\qquad\qquad\qquad \forall i \in [n].
\end{aligned}
$$

# Refinement under Cardinality Constraints

Common case in practice:

- Relations have different sizes
- Small-size projections of relations may be added to the join query

Add relation sizes into the linear program that computes the result size of a join query $Q(\boldsymbol{A}_1 \cup \cdots \cup \boldsymbol{A}_n) = R_1(\boldsymbol{A}_1), \ldots, R_n(\boldsymbol{A}_n)$:

$$
\begin{aligned}
\text{minimize} \quad & N^{\sum_{i \in [n]} x_{R_i}} \\
\text{subject to} \quad & \textstyle\sum_{i:\text{edge } R_i \text{ covers node } A} x_{R_i} \geq 1 \quad \forall A \in \bigcup_{j \in [n]} \boldsymbol{A}_j, \\
& x_{R_i} \geq 0 \qquad\qquad\qquad\qquad \forall i \in [n].
\end{aligned}
$$

Assumption: All relations have the same size $N$.

# Refinement under Cardinality Constraints

Common case in practice:

- Relations have different sizes
- Small-size projections of relations may be added to the join query

Add relation sizes into the linear program that computes the result size of a join query $Q(\boldsymbol{A}_1 \cup \cdots \cup \boldsymbol{A}_n) = R_1(\boldsymbol{A}_1), \ldots, R_n(\boldsymbol{A}_n)$:

$$\begin{aligned} \text{minimize} \quad & \prod_{i \in [n]} N^{x_i} \\ \text{subject to} \quad & \sum_{i:\text{edge } R_i \text{ covers node } A} x_{R_i} \geq 1 \quad \forall A \in \bigcup_{j \in [n]} \boldsymbol{A}_j, \\ & x_{R_i} \geq 0 \qquad\qquad\qquad \forall i \in [n]. \end{aligned}$$

Assumption: All relations have the same size $N$.

# Refinement under Cardinality Constraints

Common case in practice:

- Relations have different sizes
- Small-size projections of relations may be added to the join query

Add relation sizes into the linear program that computes the result size of a join query $Q(\boldsymbol{A}_1 \cup \cdots \cup \boldsymbol{A}_n) = R_1(\boldsymbol{A}_1), \ldots, R_n(\boldsymbol{A}_n)$:

$$\begin{aligned} \text{minimize} \quad & \prod_{i \in [n]} N_i^{x_i} \\ \text{subject to} \quad & \sum_{i:\text{edge } R_i \text{ covers node } A} x_{R_i} \geq 1 \quad \forall A \in \bigcup_{j \in [n]} \boldsymbol{A}_j, \\ & x_{R_i} \geq 0 \quad\quad\quad \forall i \in [n]. \end{aligned}$$

Assumption: Relation $R_i$ has size $N_i$, $\forall i \in [n]$.

# Size Bounds for Factorized Representations of Join Results

# Recall the Itemized Customer Orders Example

Orders (O for short)

| customer | day | dish |
|---|---|---|
| Elise | Monday | burger |
| Elise | Friday | burger |
| Steve | Friday | hotdog |
| Joe | Friday | hotdog |

Dish (D for short)

| dish | item |
|---|---|
| burger | patty |
| burger | onion |
| burger | bun |
| hotdog | bun |
| hotdog | onion |
| hotdog | sausage |

Items (I for short)

| item | price |
|---|---|
| patty | 6 |
| onion | 2 |
| bun | 2 |
| sausage | 4 |

Consider the natural join of the above relations:

O(customer, day, dish), D(dish, item), I(item, price)

| customer | day | dish | item | price |
|---|---|---|---|---|
| Elise | Monday | burger | patty | 6 |
| Elise | Monday | burger | onion | 2 |
| Elise | Monday | burger | bun | 2 |
| Elise | Friday | burger | patty | 6 |
| Elise | Friday | burger | onion | 2 |
| Elise | Friday | burger | bun | 2 |
| ... | ... | ... | ... | ... |

# Factor Out Common Data Blocks

O(customer, day, dish), D(dish, item), I(item, price)

| customer | day | dish | item | price |
|---|---|---|---|---|
| Elise | Monday | burger | patty | 6 |
| Elise | Monday | burger | onion | 2 |
| Elise | Monday | burger | bun | 2 |
| Elise | Friday | burger | patty | 6 |
| Elise | Friday | burger | onion | 2 |
| Elise | Friday | burger | bun | 2 |
| ... | ... | ... | ... | ... |

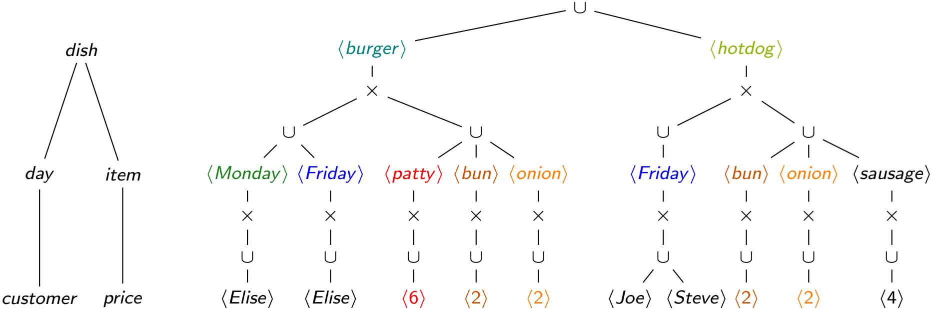
The listing representation of the above query result is:

$$\begin{array}{lllllllllll}
\langle \mathit{Elise} \rangle & \times & \langle \mathit{Monday} \rangle & \times & \langle \mathit{burger} \rangle & \times & \langle \text{patty} \rangle & \times & \langle 6 \rangle & \cup \\
\langle \mathit{Elise} \rangle & \times & \langle \mathit{Monday} \rangle & \times & \langle \mathit{burger} \rangle & \times & \langle \text{onion} \rangle & \times & \langle 2 \rangle & \cup \\
\langle \mathit{Elise} \rangle & \times & \langle \mathit{Monday} \rangle & \times & \langle \mathit{burger} \rangle & \times & \langle \text{bun} \rangle & \times & \langle 2 \rangle & \cup \\
\langle \mathit{Elise} \rangle & \times & \langle \mathit{Friday} \rangle & \times & \langle \mathit{burger} \rangle & \times & \langle \text{patty} \rangle & \times & \langle 6 \rangle & \cup \\
\langle \mathit{Elise} \rangle & \times & \langle \mathit{Friday} \rangle & \times & \langle \mathit{burger} \rangle & \times & \langle \text{onion} \rangle & \times & \langle 2 \rangle & \cup \\
\langle \mathit{Elise} \rangle & \times & \langle \mathit{Friday} \rangle & \times & \langle \mathit{burger} \rangle & \times & \langle \text{bun} \rangle & \times & \langle 2 \rangle & \cup \ldots
\end{array}$$

It uses relational product ($\times$), union ($\cup$), and data (singleton relations).

- The attribute names are not shown to avoid clutter.

# This is How A Factorized Join Looks Like!



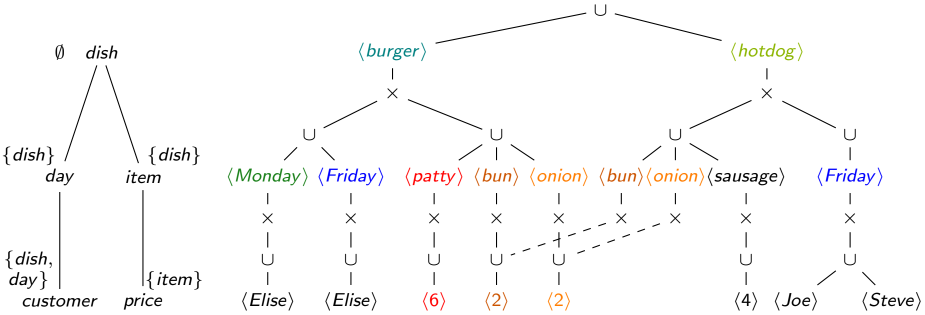

Var order

Factorized representation of the join result

There are several *algebraically equivalent* factorized representations defined:

- by distributivity of product over union and their commutativity;
- as groundings of variable orders.

# .. Now with Further Compression using Caching

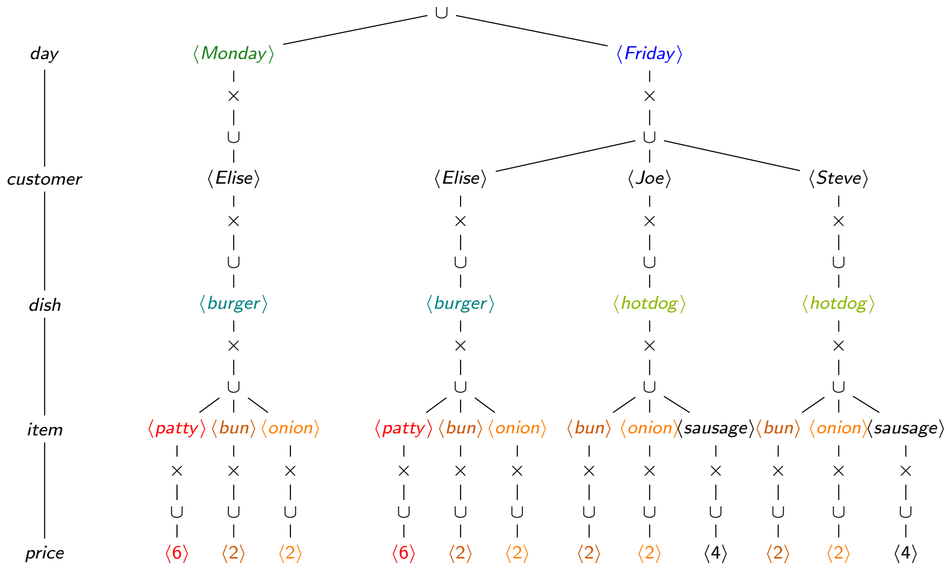



Observation:

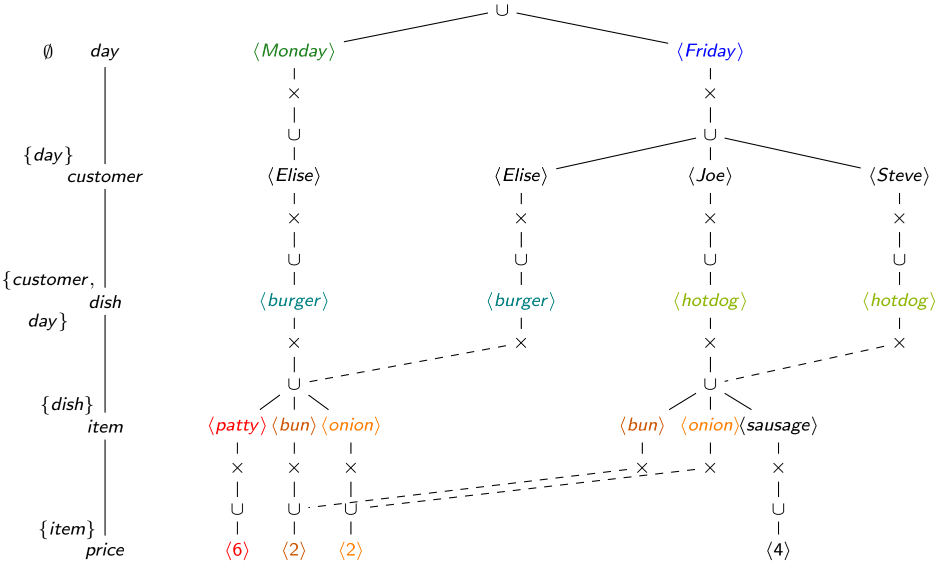
- `price` is under `item`, which is under `dish`, but only *depends* on `item`,
- .. so the same `price` appears under an `item` *regardless* of the `dish`.

Idea: *Cache* `price` for a specific `item` and avoid repetition!

# Same Data, Different Factorization



# .. and Further Compressed using Caching



# Which factorization should we choose?

The *size* of a factorization is the number of its values.

Example:

$$
\begin{aligned}
F_1 =& \big(\langle 1\rangle \cup \cdots \cup \langle n\rangle\big) \times \big(\langle 1\rangle \cup \cdots \cup \langle m\rangle\big)\\
F_2 =& \langle 1\rangle \times \langle 1\rangle \cup \cdots \cup \langle 1\rangle \times \langle m\rangle\\
& \cup \cdots \cup\\
& \langle n\rangle \times \langle 1\rangle \cup \cdots \cup \langle n\rangle \times \langle m\rangle.
\end{aligned}
$$

- $F_1$ is factorized, $F_2$ is a listing representation
- $F_1 \equiv F_2$
- **BUT** $|F_1| = m + n \ll |F_2| = m * n$.

**How much space does factorization save over the listing representation?**

# Size Bounds for Join Results

Given a join query $Q$, for any database of size $N$, the join result admits

- a listing representation of size $O(N^{\rho^*(Q)})$. [LW49,A81,BT95,AGM08]

# Size Bounds for Join Results

Given a join query $Q$, for any database of size $N$, the join result admits

- a listing representation of size $O(N^{\rho^*(Q)})$. [LW49,A81,BT95,AGM08]
- a factorization *without caching* of size $O(N^{s(Q)})$. [OZ12]

# Size Bounds for Join Results

Given a join query $Q$, for any database of size $N$, the join result admits

- a listing representation of size $O(N^{\rho^*(Q)})$. [LW49,A81,BT95,AGM08]
- a factorization *without caching* of size $O(N^{s(Q)})$. [OZ12]
- a factorization *with caching* of size $O(N^{fhtw(Q)})$. [OZ15]

# Size Bounds for Join Results

Given a join query $Q$, for any database of size $N$, the join result admits

- a listing representation of size $O(N^{\rho^*(Q)})$. [LW49,A81,BT95,AGM08]

- a factorization *without caching* of size $O(N^{s(Q)})$. [OZ12]

- a factorization *with caching* of size $O(N^{fhtw(Q)})$. [OZ15]

$$1 \leq fhtw(Q) \underbrace{\leq}_{\text{up to } \log|Q|} s(Q) \underbrace{\leq}_{\text{up to } |Q|} \rho^*(Q) \leq |Q|$$

- $|Q|$ is the number of relations in $Q$
- $\rho^*(Q)$ is the fractional edge cover number of $Q$
- $s(Q)$ is the factorization width of $Q$
- $fhtw(Q)$ is the fractional hypertree width of $Q$ [M10]

# Size Bounds for Join Results

Given a join query $Q$, for any database of size $N$, the join result admits

- a listing representation of size $O(N^{\rho^*(Q)})$. [LW49,A81,BT95,AGM08]
- a factorization *without caching* of size $O(N^{s(Q)})$. [OZ12]
- a factorization *with caching* of size $O(N^{fhtw(Q)})$. [OZ15]

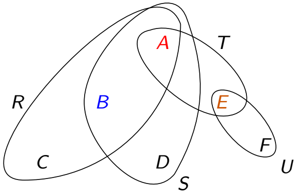
These size bounds are asymptotically tight!

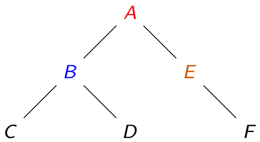
- **Best possible size bounds** for factorized representations over variable orders of $Q$ and for listing representation, *but not database optimal!*

  There exists arbitrarily large databases for which

  - the listing representation has size $\Omega(N^{\rho^*(Q)})$
  - the factorization with/without caching over *any variable order* of $Q$ has size $\Omega(N^{s(Q)})$ and $\Omega(N^{fhtw(Q)})$ respectively.

# Example: The Factorization Width $s$




The structure of the factorization over the above variable order $\Delta$:

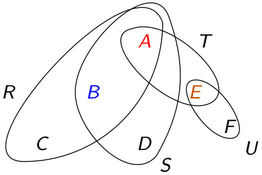
$$\bigcup_{a \in \mathbf{A}} \Big( \langle a \rangle \times \bigcup_{b \in \mathbf{B}} \Big( \langle b \rangle \times \Big( \bigcup_{c \in C} \langle c \rangle \Big) \times \Big( \bigcup_{d \in D} \langle d \rangle \Big) \Big) \times \bigcup_{e \in \mathbf{E}} \Big( \langle e \rangle \times \Big( \bigcup_{f \in F} \langle f \rangle \Big) \Big) \Big)$$

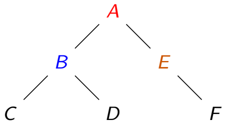
The number of values for a variable is dictated by the number of valid tuples of values for its ancestors in $\Delta$:

- One value $\langle f \rangle$ for each tuple $(a, e, f)$ in the join result.

Size of factorization = sum of sizes of results of **subqueries along paths**.

# Example: The Factorization Width $s$




- The factorization width for $\Delta$ is the largest $\rho^*$ over subqueries defined by root-to-leaf paths in $\Delta$
- $s(Q)$ is the minimum factorization width over all variable orders of $Q$

In our example:

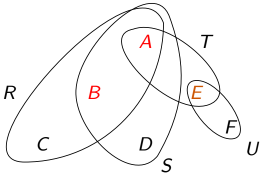
- Path $A$–$E$–$F$ has fractional edge cover number 2.
  $\Rightarrow$ The number of $F$-values is $\leq N^2$, but can be $\sim N^2$.
- All other root-to-leaf paths have fractional edge cover number 1.
  $\Rightarrow$ The number of other values is $\leq N$.

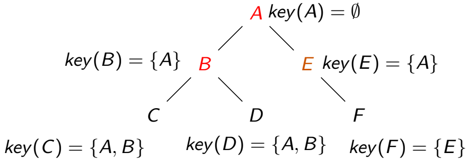
$s(Q) = 2$ $\Rightarrow$ Factorization size is $O(N^2)$

Recall that $\rho^*(Q) = 3$ $\Rightarrow$ Listing representation size is $O(N^3)$

# Example: The Fractional Hypertree Width *fhtw*

Idea: Avoid repeating identical expressions, store them once and use pointers.




$$\bigcup_{a \in \mathbf{A}} [\langle a \rangle \times \cdots \times \bigcup_{e \in \mathbf{E}} (\langle e \rangle \times (\bigcup_{f \in F} \langle f \rangle))]$$

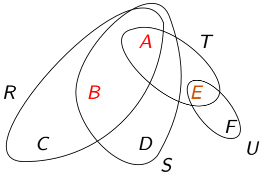
Observation:

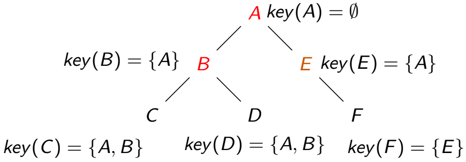
- Variable $F$ only depends on **E** and not on **A**: $key(F) = \{\mathbf{E}\}$
- A value $\langle e \rangle$ maps to the same union $\bigcup_{(e,f)\in U} \langle f \rangle$ regardless of its pairings with **A**-values.

  $\Rightarrow$ Define $U_e = \bigcup_{(e,f)\in U} \langle f \rangle$ once for each value $\langle e \rangle$ and reuse it

# Example: The Fractional Hypertree Width *fhtw*

Idea: Avoid repeating identical expressions, store them once and use pointers.




A factorization with caching would be:

$$\bigcup_{a \in \mathbf{A}} \left[ \langle a \rangle \times \cdots \times \bigcup_{e \in \mathbf{E}} \left( \langle e \rangle \times U_e \right) \right]; \qquad \left\{ U_e = \bigcup_{(e,f) \in U} \langle f \rangle \right\}$$

- *fhtw* for $\Delta$ is the largest $\rho^*(Q_{key(X) \cup \{X\}})$ over subqueries $Q_{key(X) \cup \{X\}}$ defined by the variables $key(X) \cup \{X\}$ for each variable $X$ in $\Delta$
- $fhtw(Q)$ is the minimum *fhtw* over all variable orders of $Q$

In our example: $fhtw(Q) = 1 < s(Q) = 2 < \rho^*(Q) = 3$.

# Alternative Characterizations of *fhtw*

The fractional hypertree width *fhtw* has been originally defined for hypertree decompositions. [M10]

- Given a join query $Q$.
- Let $\mathbf{T}$ be the set of hypertree decompositions of the hypergraph of $Q$.

$$\boxed{\mathit{fhtw}(Q) = \min_{(T,\chi)\in\mathbf{T}} \max_{n\in T} \rho^*(Q_{\chi(n)})}$$

# Alternative Characterizations of *fhtw*

The fractional hypertree width *fhtw* has been originally defined for hypertree decompositions. [M10]

- Given a join query $Q$.
- Let $\mathbf{T}$ be the set of hypertree decompositions of the hypergraph of $Q$.

$$\boxed{fhtw(Q) = \min_{(T,\chi)\in\mathbf{T}} \max_{n\in T} \rho^*(Q_{\chi(n)})}$$

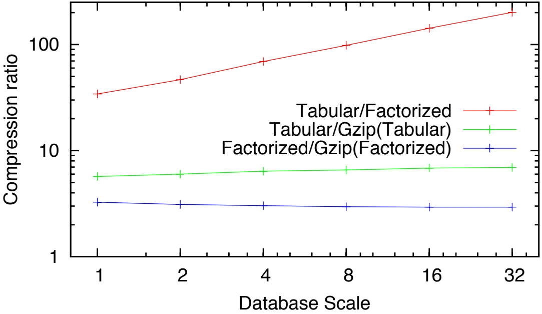
Alternative characterization of the fractional hypertree width *fhtw* using the mapping between hypertree decompositions and variable orders [OZ15]

- Given a join query $Q$.
- Let **VO** be the set of variable orders of $Q$.

$$\boxed{fhtw(Q) = \min_{(F,key)\in\mathbf{VO}} \max_{v\in F} \rho^*(Q_{key(v)\cup\{v\}})}$$

# Compression by Factorization in Practice

# Compression Contest: Factorized vs. Zipped Relations




Result of query $\text{Orders} \bowtie \text{Dish} \bowtie \text{Items}$ [BKOZ13]

- Tabular = listing representation in CSV text format
- Gzip (compression level 6) outputs binary format
- Factorized representation in text format (each digit takes one character)

Observations:

- Gzip does not exploit distant repetitions!
- Factorizations can be arbitrarily more succinct than gzipped relations.
- Gzipping factorizations improves the compression by 3x.

# Factorization Gains in Practice (1/4)

Retailer dataset used for LogicBlox analytics

- Relations: Inventory (84M), Sales (1.5M), Clearance (368K), Promotions (183K), Census (1K), Location (1K).
- Compression factors (caching not used):
  - **26.61x** for natural join of Inventory, Census, Location.
  - **159.59x** for natural join of Inventory, Sales, Clearance, Promotions

# Factorization Gains in Practice (2/4)

LastFM public dataset

- Relations: UserArtists (93K), UserFriends (25K), TaggedArtists (186K).
- Compression factors:
  - **143.54x** for joining two copies of Userartists and Userfriends

    With caching: **982.86x**
  - **253.34x** when also joining on TaggedArtists
  - **2.53x/ 3.04x/ 924.46x** for triangle/4-clique/bowtie query on UserFriends
  - **9213.51x/ 552Kx/ $\geq$86Mx** for versions of triangle/4-clique/bowtie queries with copies for UserArtists for each UserFriend copy

# Factorization Gains in Practice (3/4)

Twitter public dataset

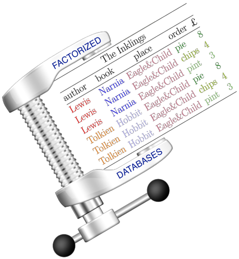
- Relation: Follower-Followee (1M)
- Compression factors:
  - **2.69x** for triangle query
  - **3.48x** for 4-clique query
  - **4918.73x** for bowtie query

# Factorization Gains in Practice (4/4)

Yelp Dataset Challenge

- Relations: Business (174K), User (1.3M), Review (5.2M), Category(667K), Attribute (1.3M)
- Compression factors:
  - **39.43x** for natural join of Business, User, Review, Attribute (with caching)
  - **185.87x** for natural join of Business, User, Review, Attribute, Category (with caching)

# Outline of Part 1: Joins




Introduction by Examples

Decompositions and Variable Orders

Size Bounds for Join Results

**Worst-Case Optimal Join Algorithms**

Further Work and References

Quiz

# How Fast Can We Compute Join Results?

Given a join query $Q$, for any database of size $N$, the join result can be computed in time

- $O(N^{\rho^*(Q)})$ as listing representation [NPRR12,V14]
- $O(N^{s(Q)})$ as factorization *without caching* [OZ15]
- $O(N^{fhtw(Q)})$ as factorization *with caching* [OZ15]

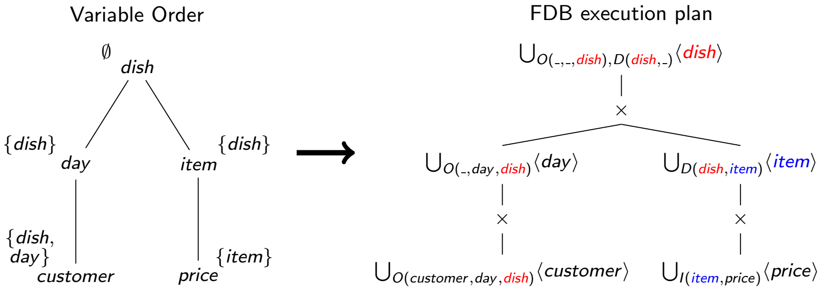
These upper bounds essentially follow the succinctness gap. They are:

- worst-case optimal (modulo $\log N$) within the given representation model
- with respect to *data* complexity
  - additional quadratic factor in the number of variables and linear factor in the number of relations in $Q$

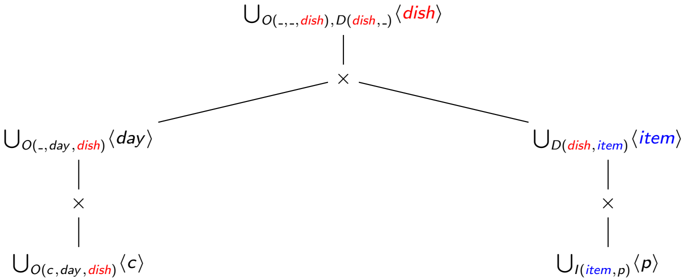
# Example: Computing the Factorized Join Result with FDB

Our join: O(customer, day, dish), D(dish, item), I(item, price)

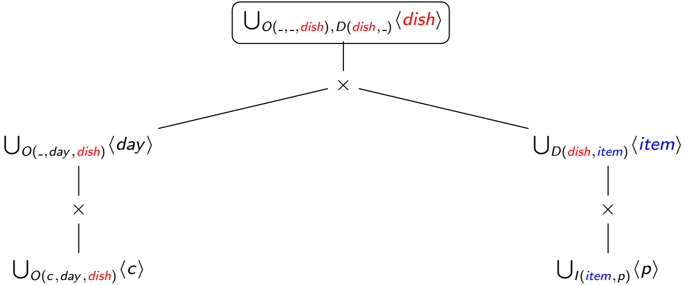
can be grounded to a factorized representation as follows:



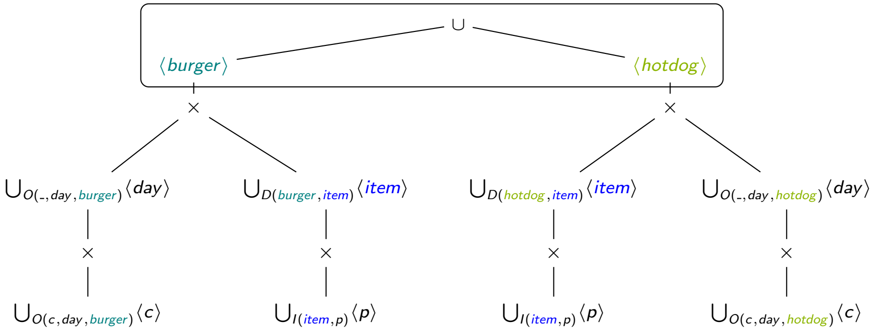
# Example: Computing the Factorized Join Result with FDB



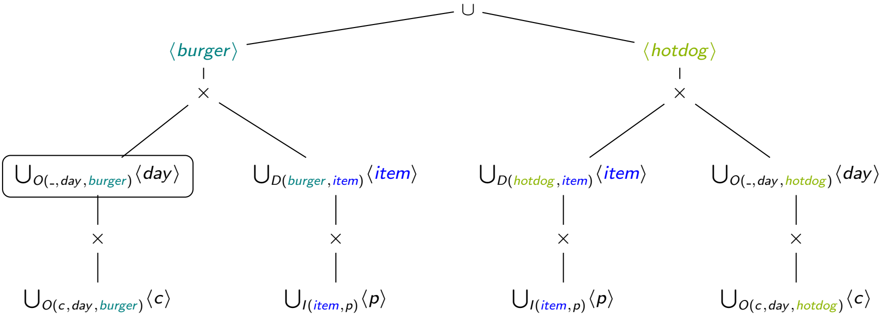
# Example: Computing the Factorized Join Result with FDB



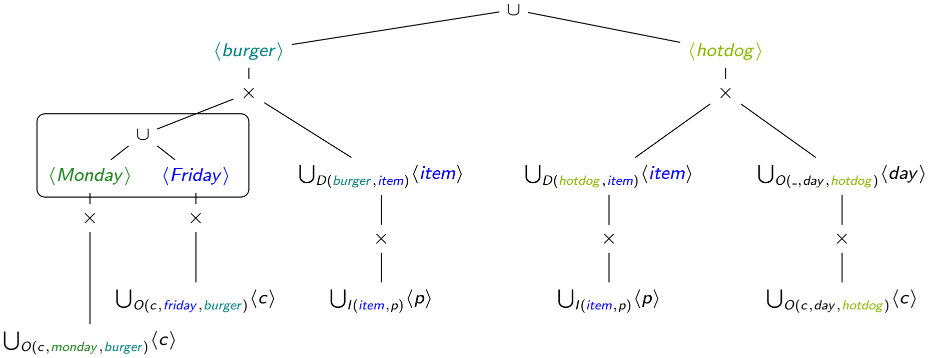
# Example: Computing the Factorized Join Result with FDB



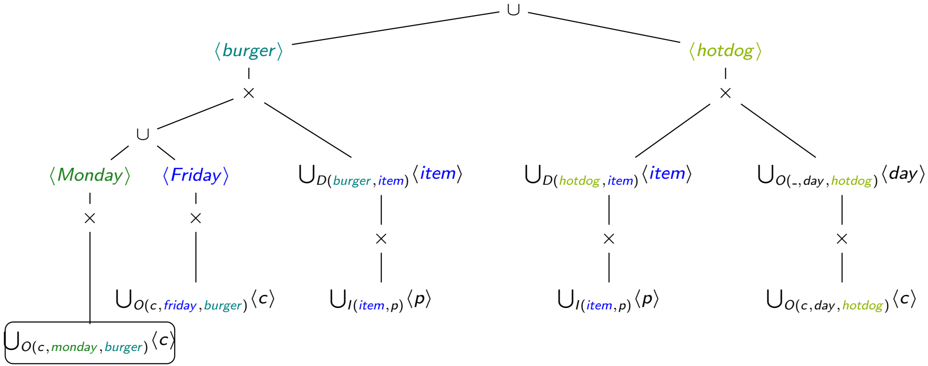
# Example: Computing the Factorized Join Result with FDB



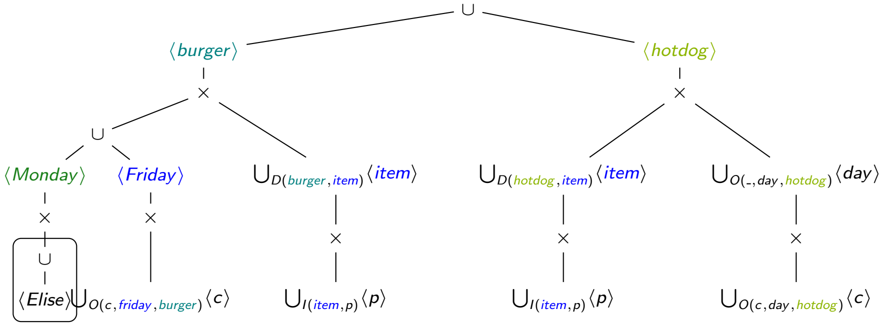
# Example: Computing the Factorized Join Result with FDB



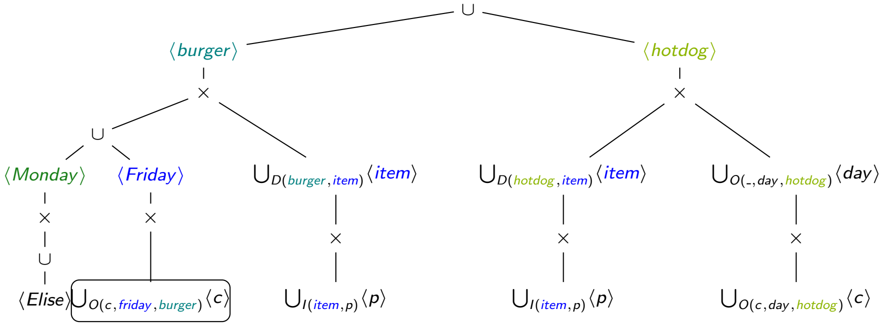
# Example: Computing the Factorized Join Result with FDB



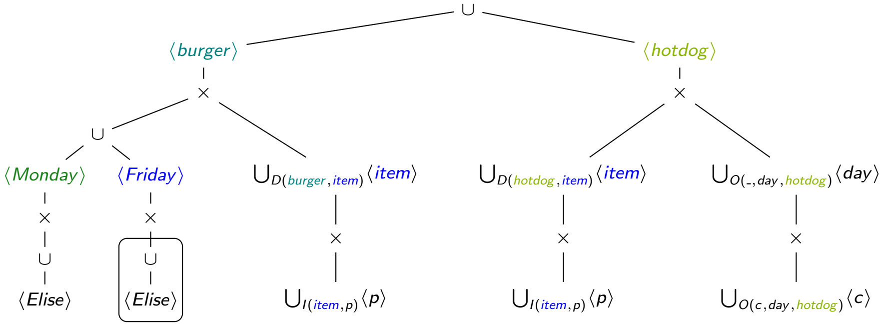
# Example: Computing the Factorized Join Result with FDB



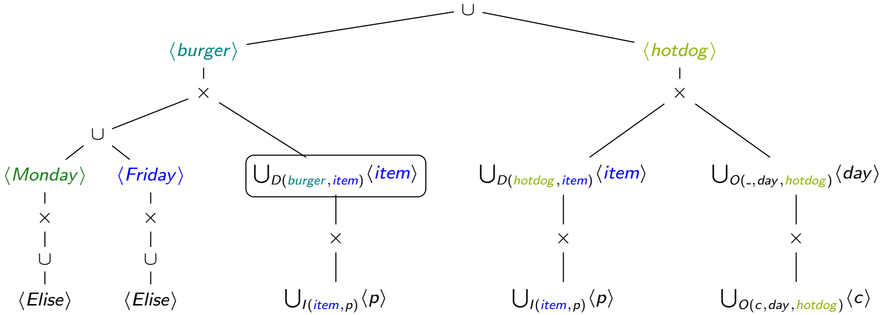
# Example: Computing the Factorized Join Result with FDB



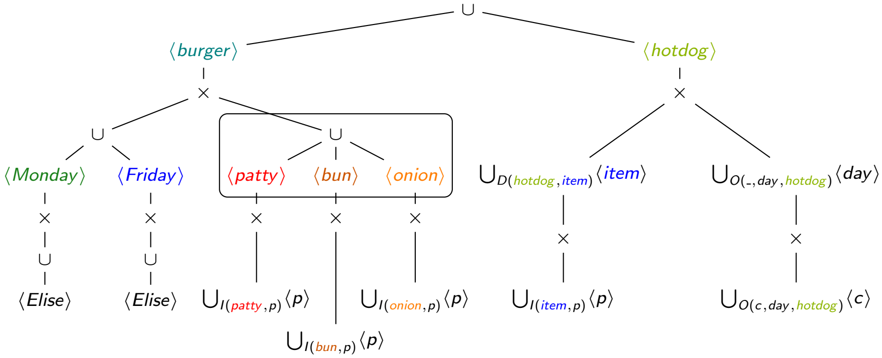
# Example: Computing the Factorized Join Result with FDB



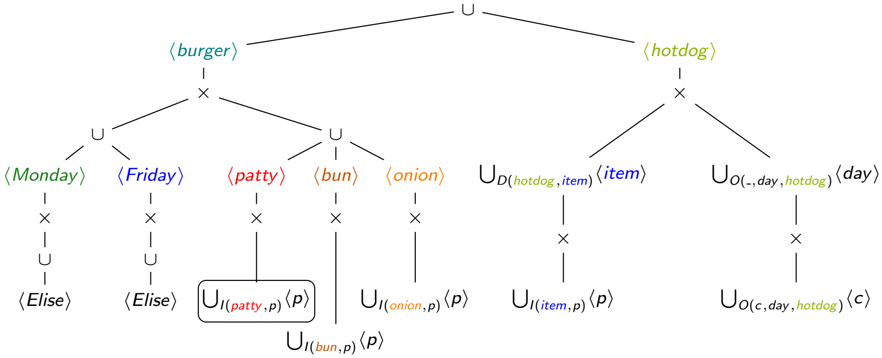
# Example: Computing the Factorized Join Result with FDB



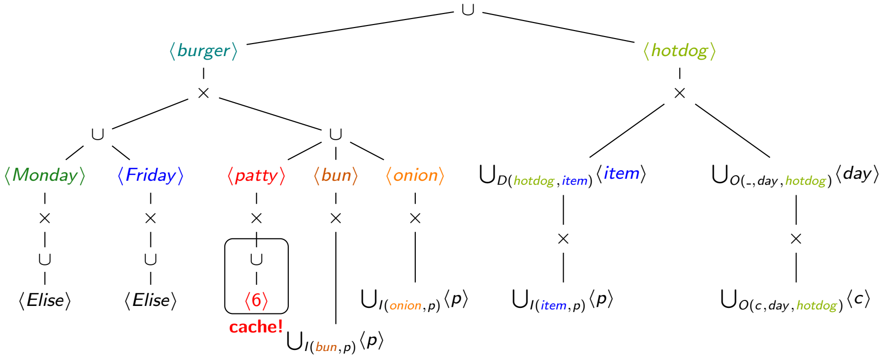
# Example: Computing the Factorized Join Result with FDB



# Example: Computing the Factorized Join Result with FDB

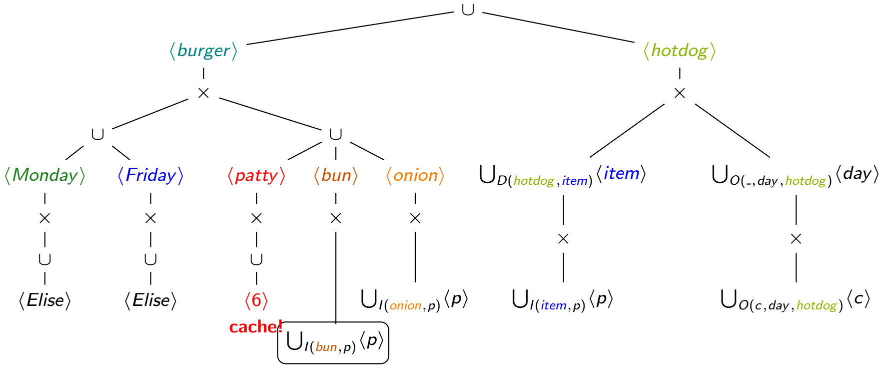


# Example: Computing the Factorized Join Result with FDB



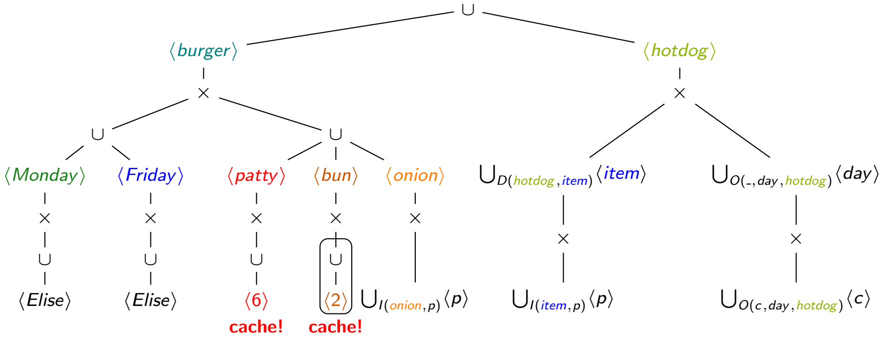

- `price` depends on `item`, but not on `dish`.
  *Cache* `prices` for specific `items`!

# Example: Computing the Factorized Join Result with FDB



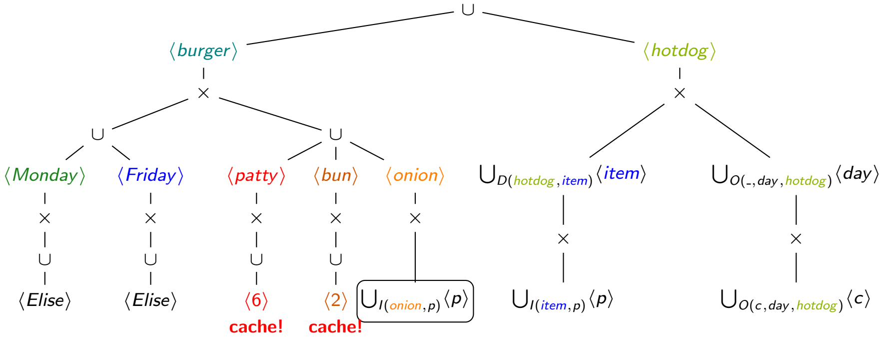

- price depends on item, but not on dish.
  *Cache* prices for specific items!

# Example: Computing the Factorized Join Result with FDB



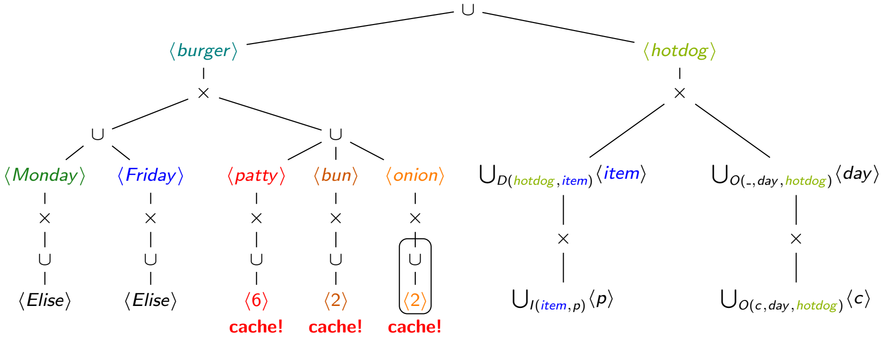

- price depends on item, but not on dish.
  *Cache* prices for specific items!

# Example: Computing the Factorized Join Result with FDB



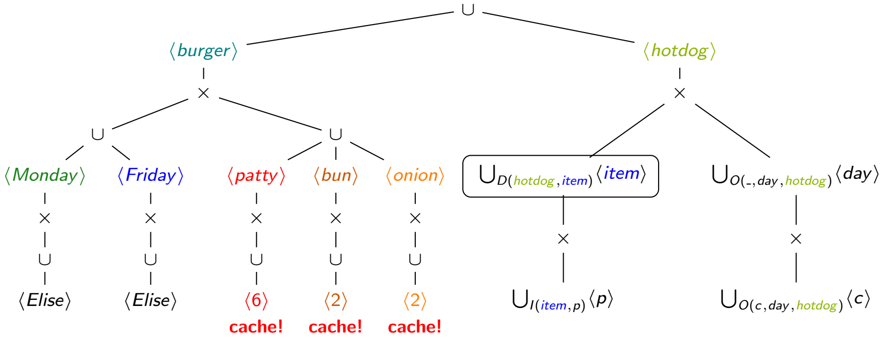

- price depends on item, but not on dish.
  *Cache* prices for specific items!

# Example: Computing the Factorized Join Result with FDB



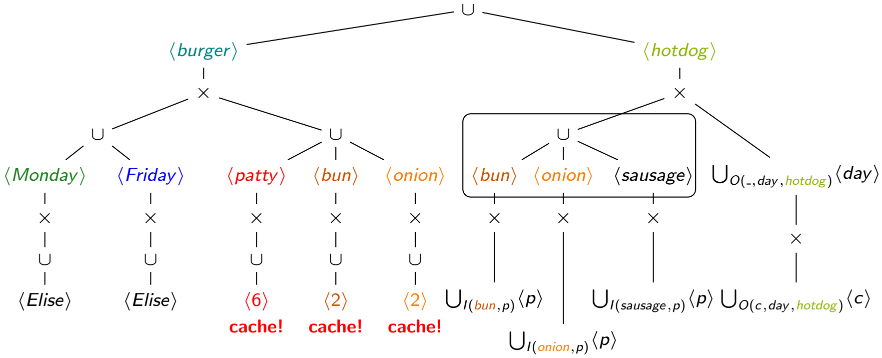

- `price` depends on `item`, but not on `dish`.
  *Cache* `prices` for specific `items`!

# Example: Computing the Factorized Join Result with FDB



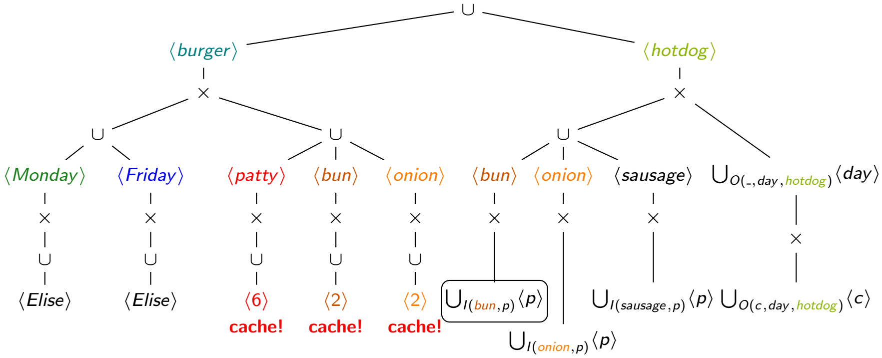

- price depends on item, but not on dish.
  *Cache* prices for specific items!

# Example: Computing the Factorized Join Result with FDB



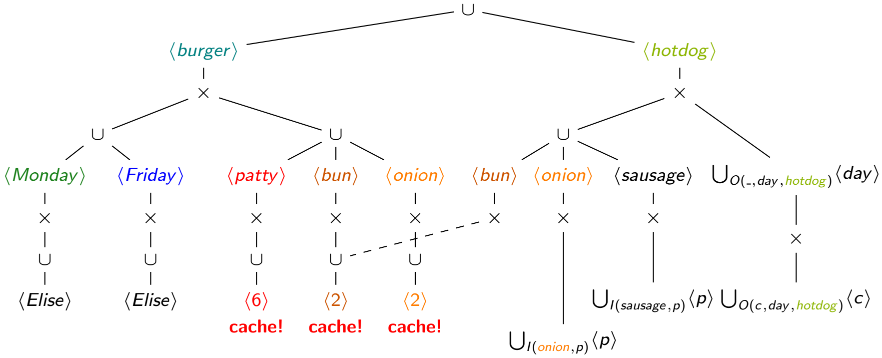

- price depends on item, but not on dish.
  *Cache* prices for specific items!

# Example: Computing the Factorized Join Result with FDB



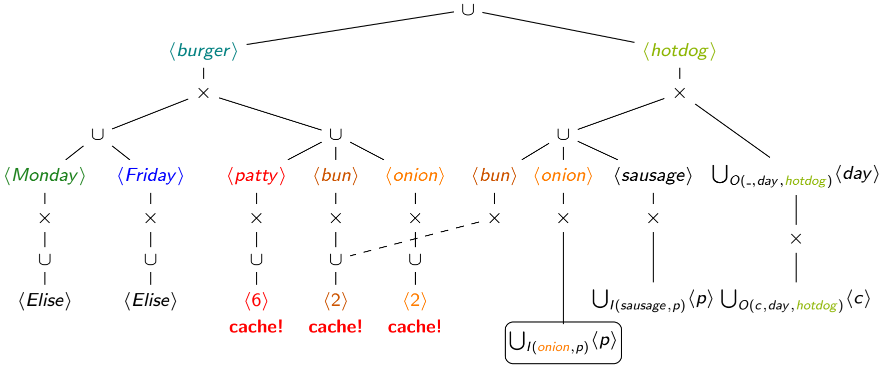

- `price` depends on `item`, but not on `dish`.
  *Cache* `prices` for specific `items`!

# Example: Computing the Factorized Join Result with FDB



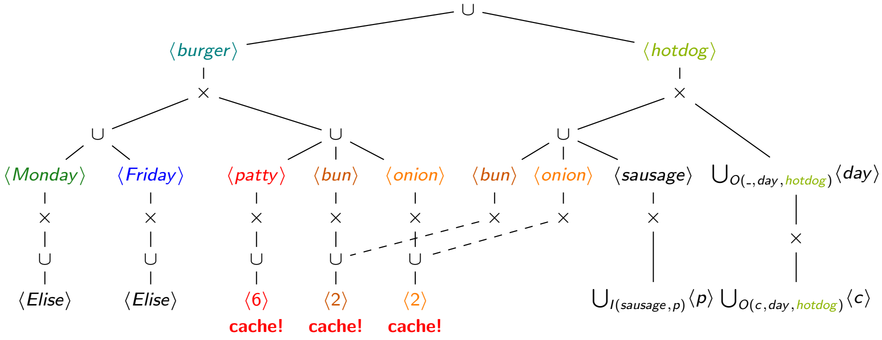

- `price` depends on `item`, but not on `dish`.
  *Cache* `prices` for specific `items`!
- Reuse cached `prices` for specific `items`!

# Example: Computing the Factorized Join Result with FDB



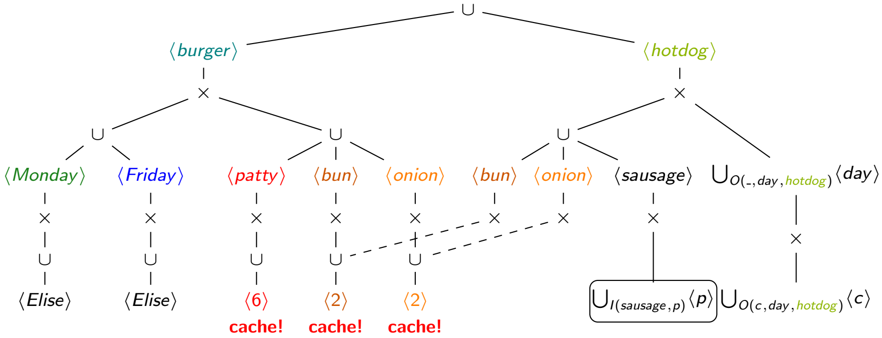

- price depends on item, but not on dish.
  *Cache* prices for specific items!
- Reuse cached prices for specific items!

# Example: Computing the Factorized Join Result with FDB



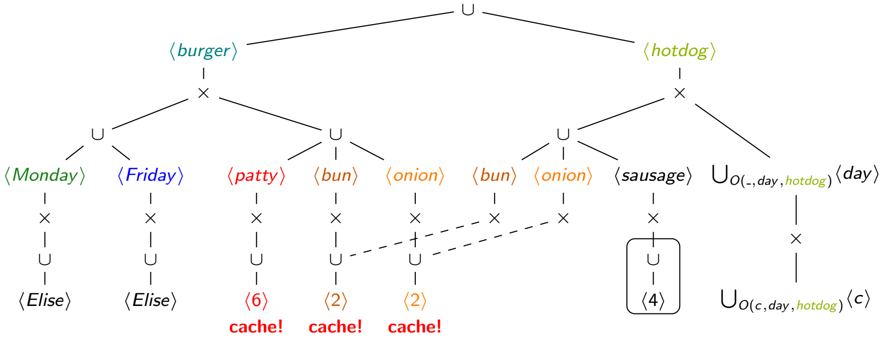

- price depends on item, but not on dish.
  *Cache* prices for specific items!
- Reuse cached prices for specific items!

# Example: Computing the Factorized Join Result with FDB



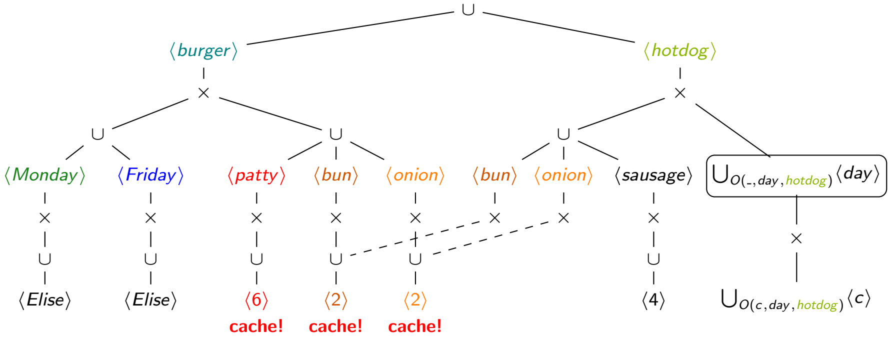

- price depends on item, but not on dish.
  *Cache* prices for specific items!
- Reuse cached prices for specific items!

# Example: Computing the Factorized Join Result with FDB



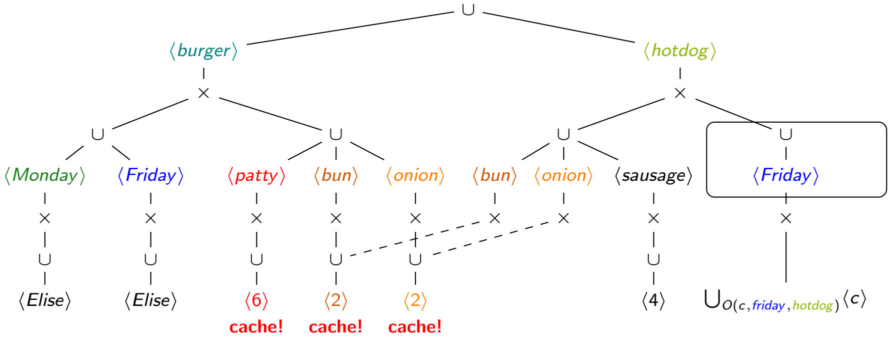

- `price` depends on `item`, but not on `dish`.
  *Cache* `prices` for specific `items`!
- Reuse cached `prices` for specific `items`!

# Example: Computing the Factorized Join Result with FDB



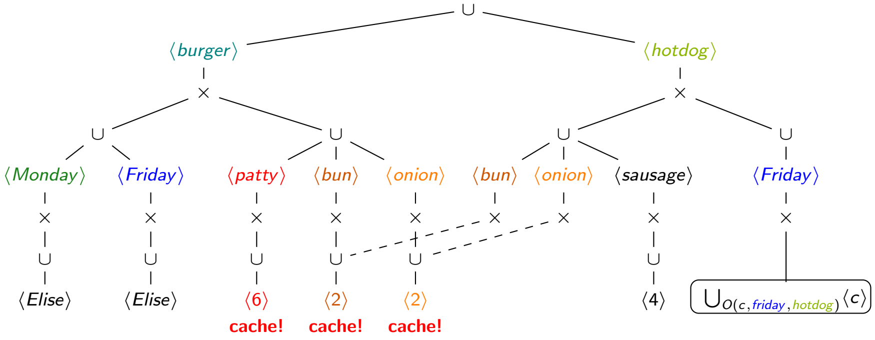

- `price` depends on `item`, but not on `dish`.
  *Cache* `prices` for specific `items`!
- Reuse cached `prices` for specific `items`!

# Example: Computing the Factorized Join Result with FDB



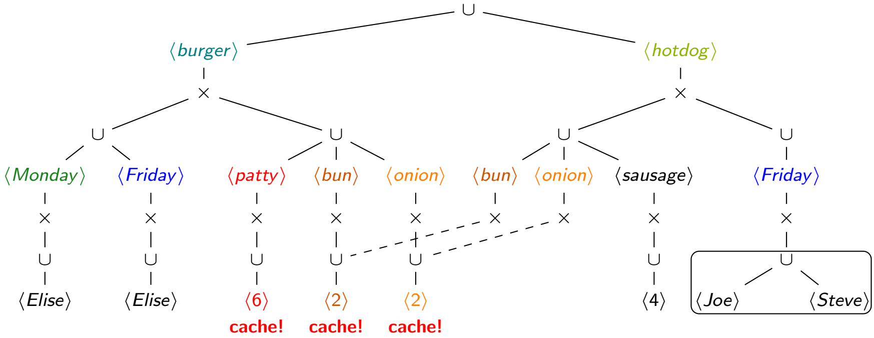

- price depends on item, but not on dish.
  *Cache* prices for specific items!
- Reuse cached prices for specific items!

# Example: Computing the Factorized Join Result with FDB



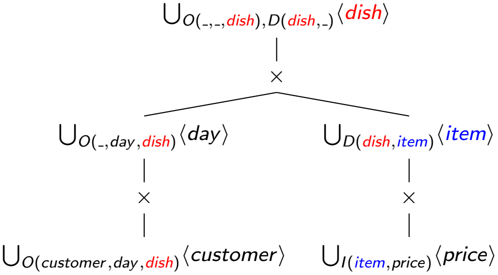

- `price` depends on `item`, but not on `dish`.
  *Cache* `prices` for specific `items`!
- Reuse cached `prices` for specific `items`!

# Example: Computing the Factorized Join Result with FDB




- price depends on item, but not on dish.
  *Cache* prices for specific items!
- Reuse cached prices for specific items!

# Example: Computing the Factorized Join Result with FDB




- Relations are sorted following any topological order of the variable order
- The intersection of relations $O$ and $D$ on *dish* takes time $O(N_{\min}\log(N_{\max}/N_{\min}))$, where $N_m = m(|\pi_{dish}O|, |\pi_{dish}D|)$.
- The remaining operations are lookups in the relations, where we first fix the *dish* value and then the *day* and *item* values.

# LeapFrog TrieJoin Algorithm

- Much acclaimed worst-case optimal join algorithm used by LogicBlox [V14]
- Computes a listing representation of the join result

  $\Rightarrow$ It does not exploit factorization
- $\approx$ Glorified multi-way sort-merge join with an efficient list intersection
- Several generalizations, e.g., Tetris, Minesweeper, and PANDA [NRR13,ANS17]

LeapFrog TrieJoin is a special case of FDB, where

- the input variable order $\Delta$ is a path

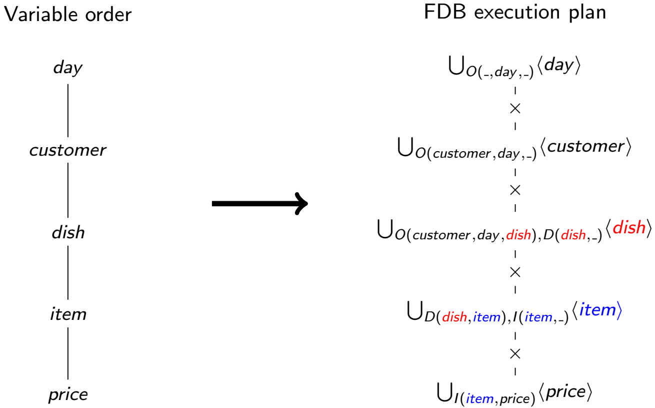
  (i.e., **no branching**)
- for each variable $A$, $key(A)$ consists of all ancestors of $A$ in $\Delta$.
  (i.e., **no caching**)

# Example: Computing the Full Join Result

The listing representation of the result of our join:

O(customer, day, dish), D(dish, item), I(item, price)

can be computed by FDB using any total variable order.



# Example: Computing the Full Join Result

$$\bigcup_{O(\_,day,\_)} \langle day \rangle$$

$$\times$$

$$\bigcup_{O(customer,day,\_)} \langle customer \rangle$$

$$\times$$

$$\bigcup_{O(customer,day,dish),D(dish,\_)} \langle dish \rangle$$

$$\times$$

$$\bigcup_{D(dish,i),I(i,\_)} \langle i \rangle$$

$$\times$$

$$\bigcup_{I(i,p)} \langle p \rangle$$

# Example: Computing the Full Join Result

$$\boxed{\bigcup_{O(\_,day,\_)} \langle day \rangle}$$

$$\times$$

$$\bigcup_{O(customer,day,\_)} \langle customer \rangle$$

$$\times$$

$$\bigcup_{O(customer,day,dish),D(dish,\_)} \langle dish \rangle$$

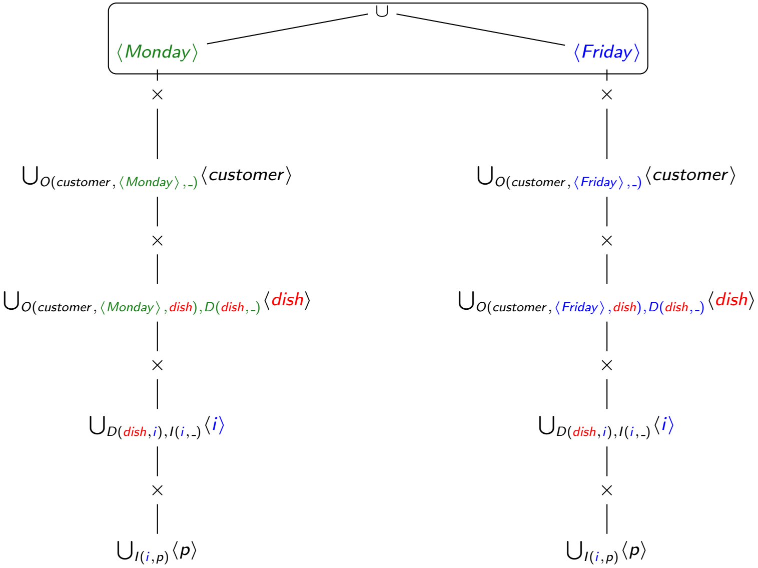
$$\times$$

$$\bigcup_{D(dish,i),I(i,\_)} \langle i \rangle$$

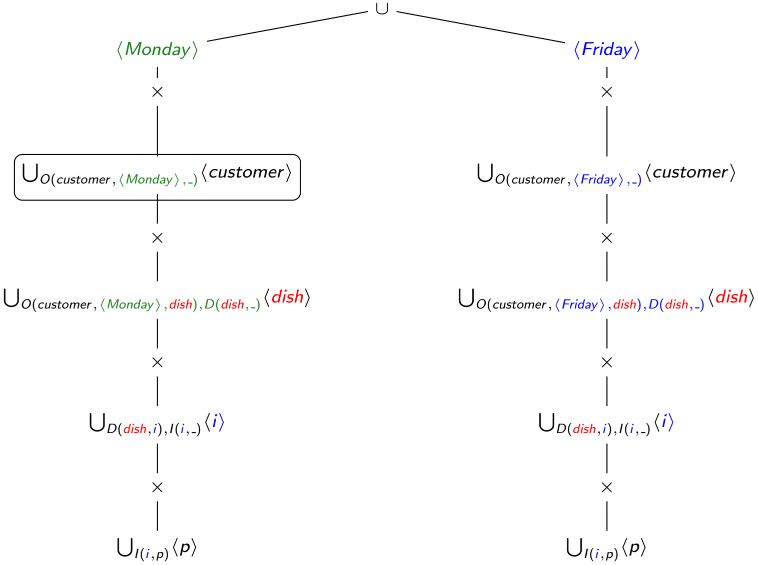
$$\times$$

$$\bigcup_{I(i,p)} \langle p \rangle$$

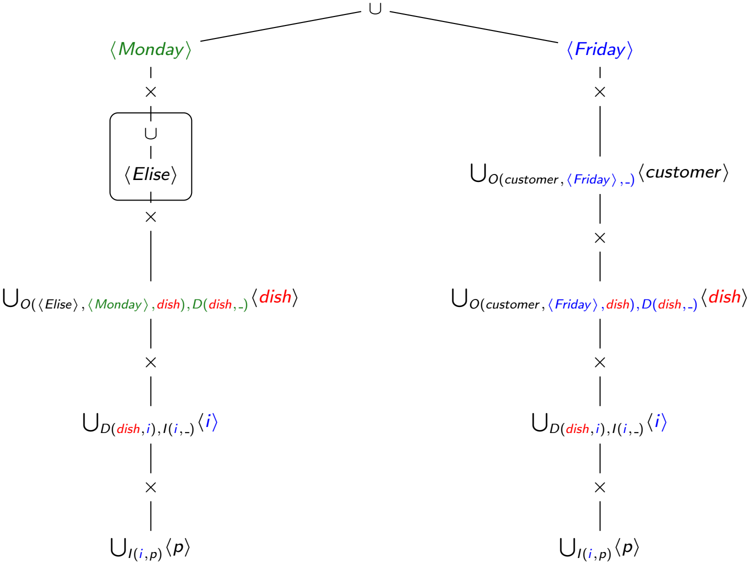
# Example: Computing the Full Join Result



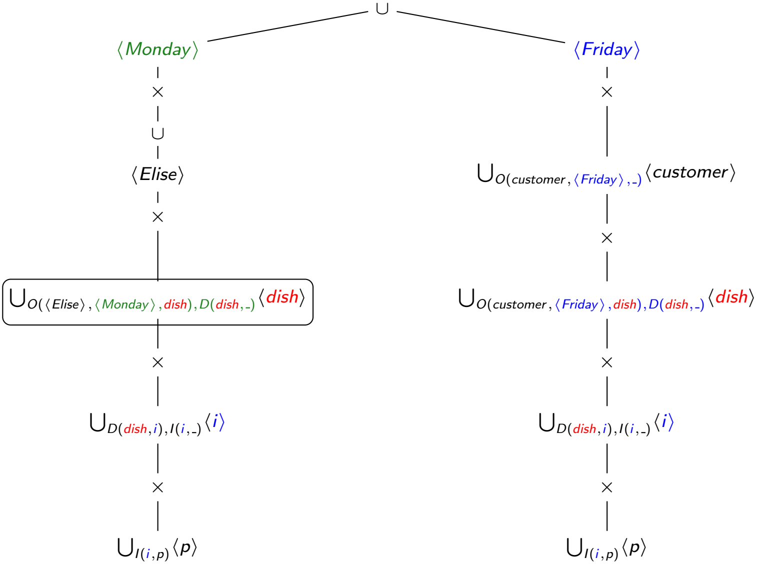
# Example: Computing the Full Join Result



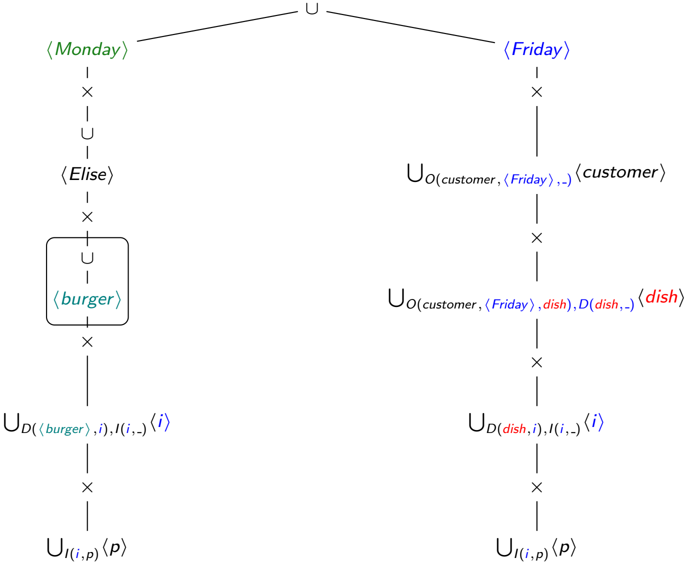
# Example: Computing the Full Join Result



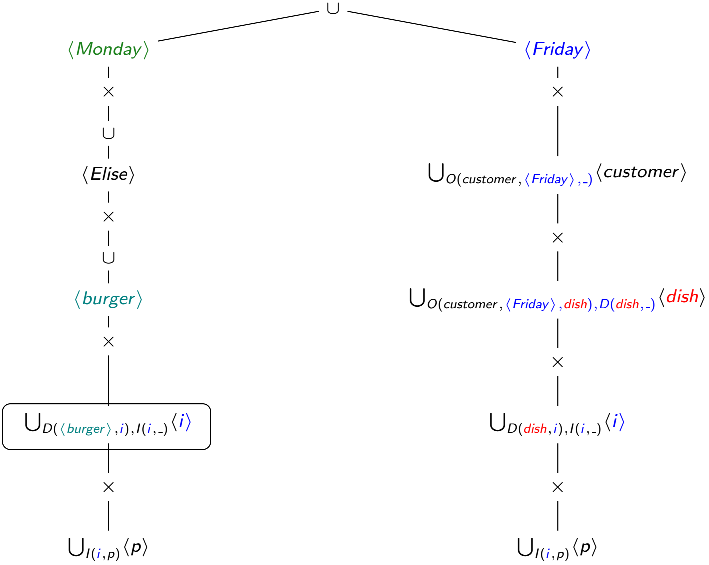
# Example: Computing the Full Join Result



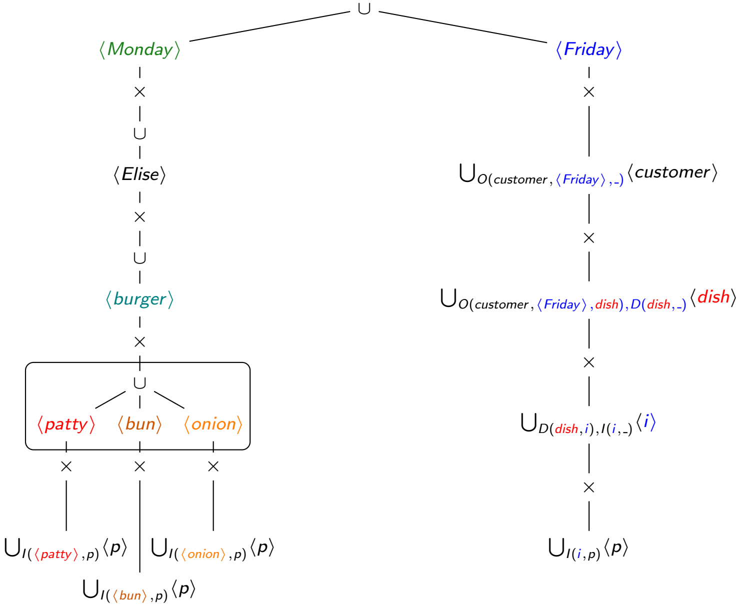
# Example: Computing the Full Join Result



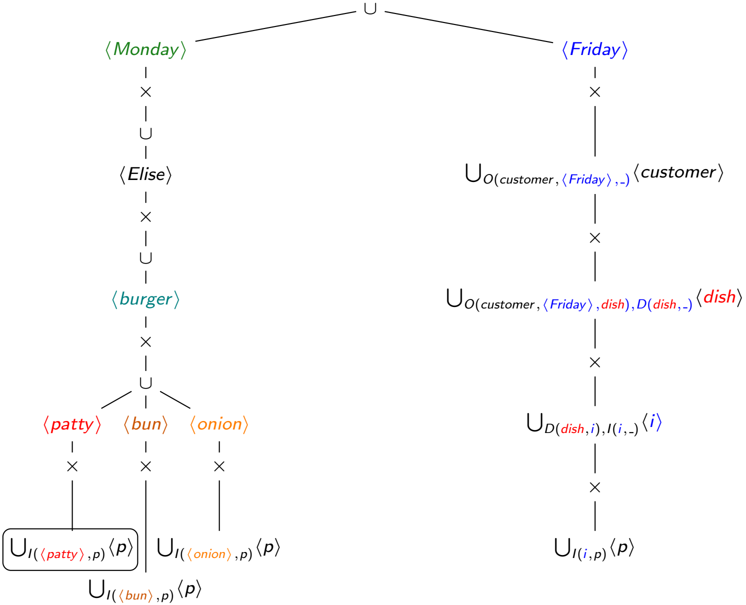
# Example: Computing the Full Join Result



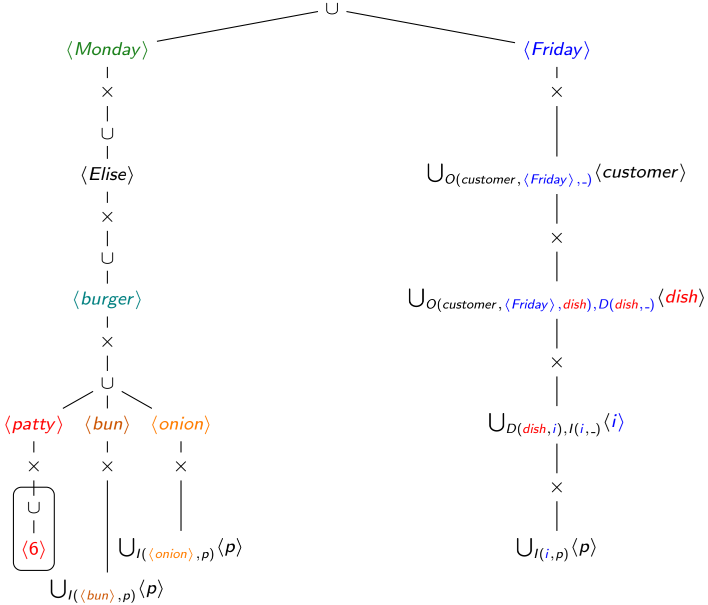
# Example: Computing the Full Join Result



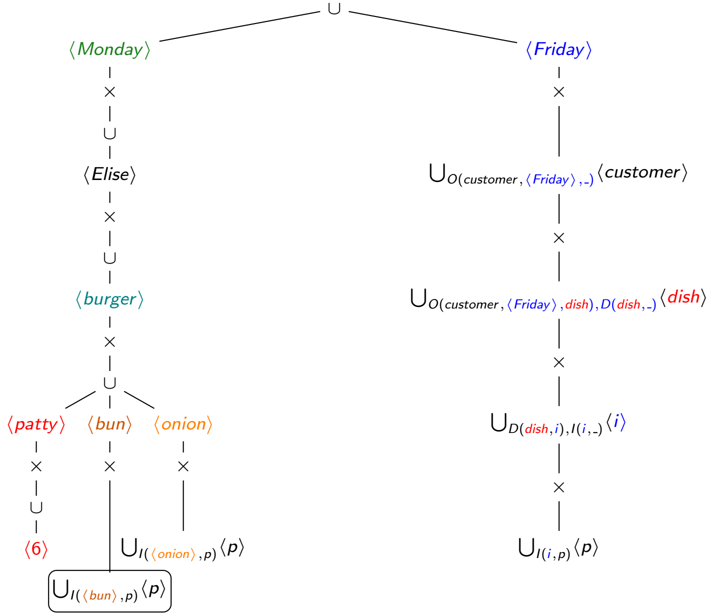
# Example: Computing the Full Join Result



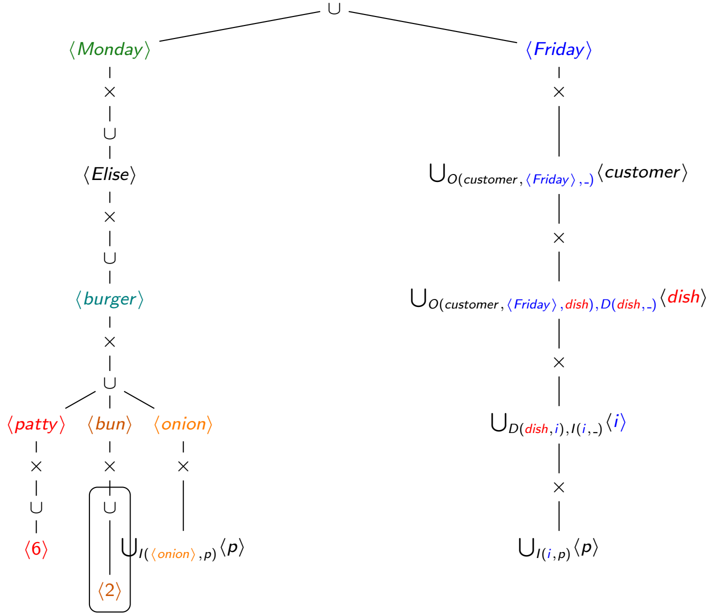
# Example: Computing the Full Join Result



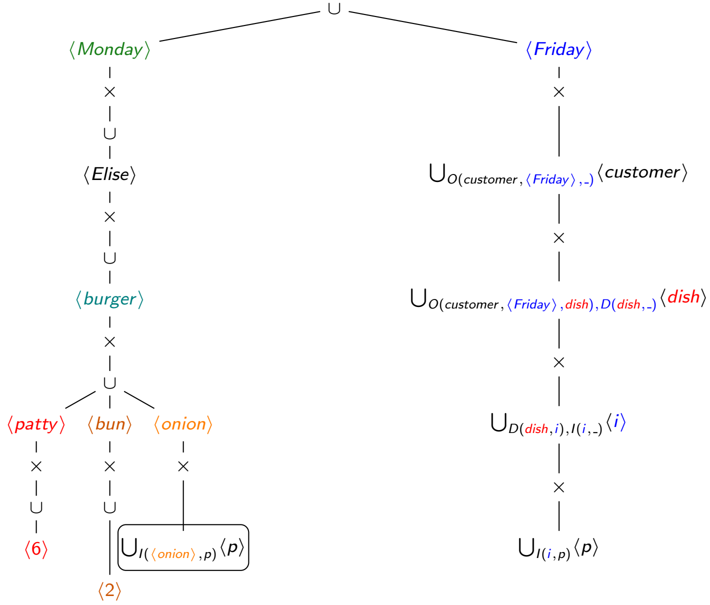
# Example: Computing the Full Join Result



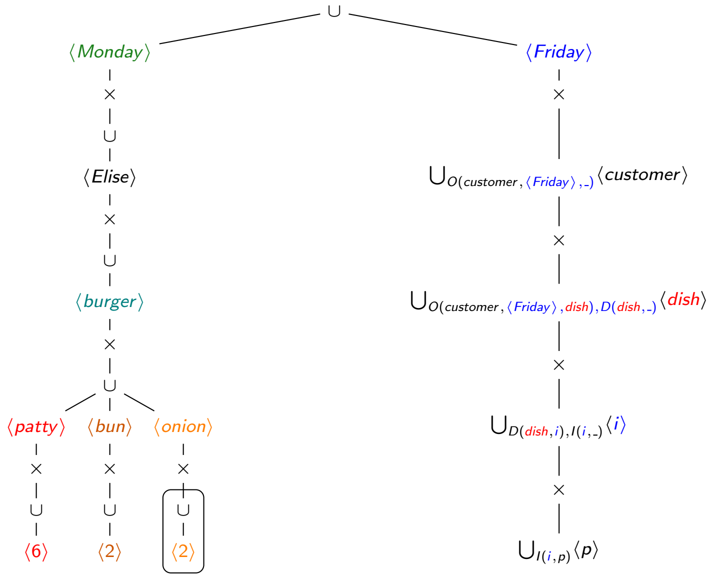
# Example: Computing the Full Join Result



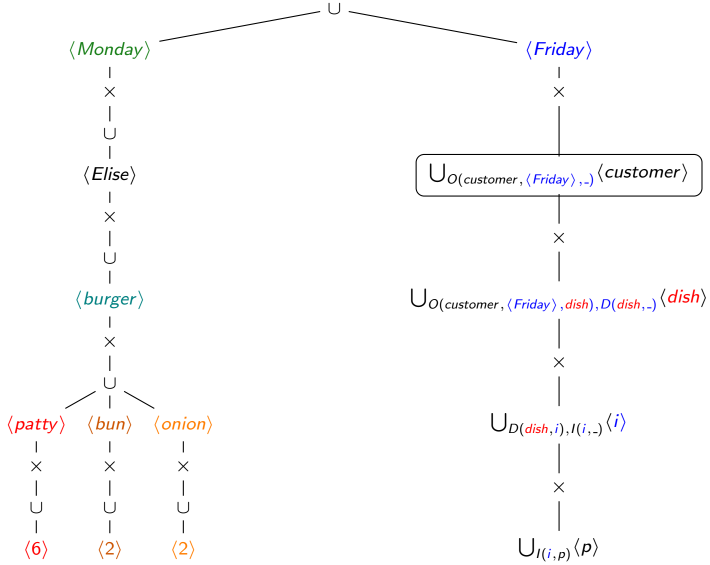
# Example: Computing the Full Join Result



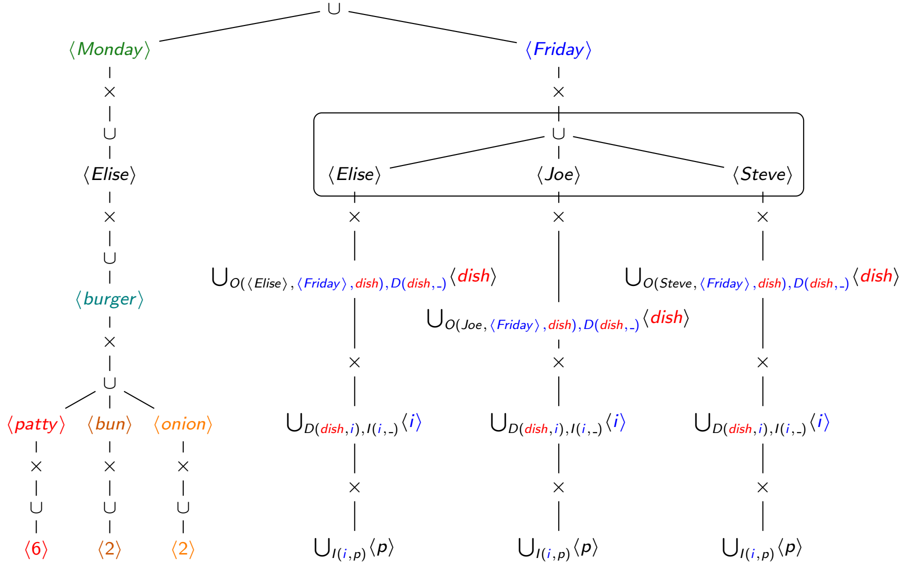
# Example: Computing the Full Join Result



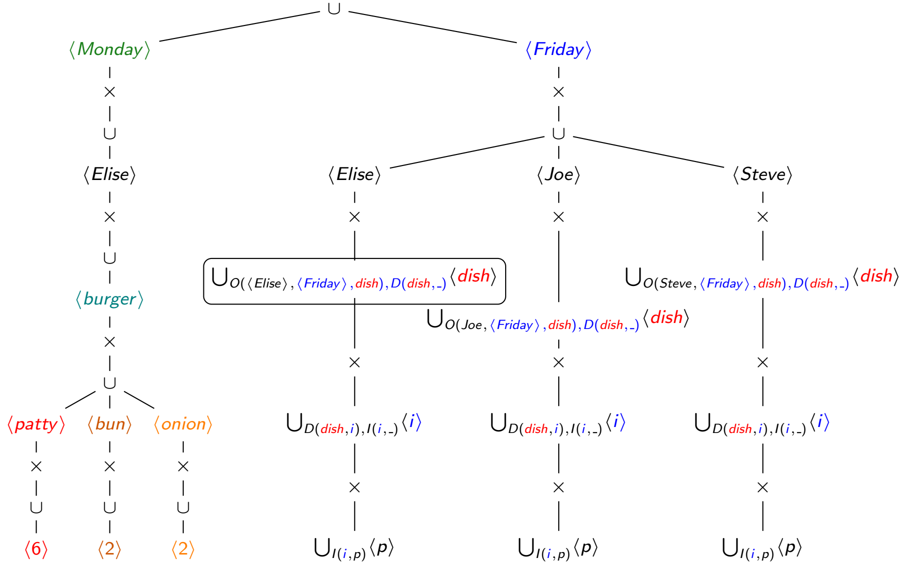
# Example: Computing the Full Join Result



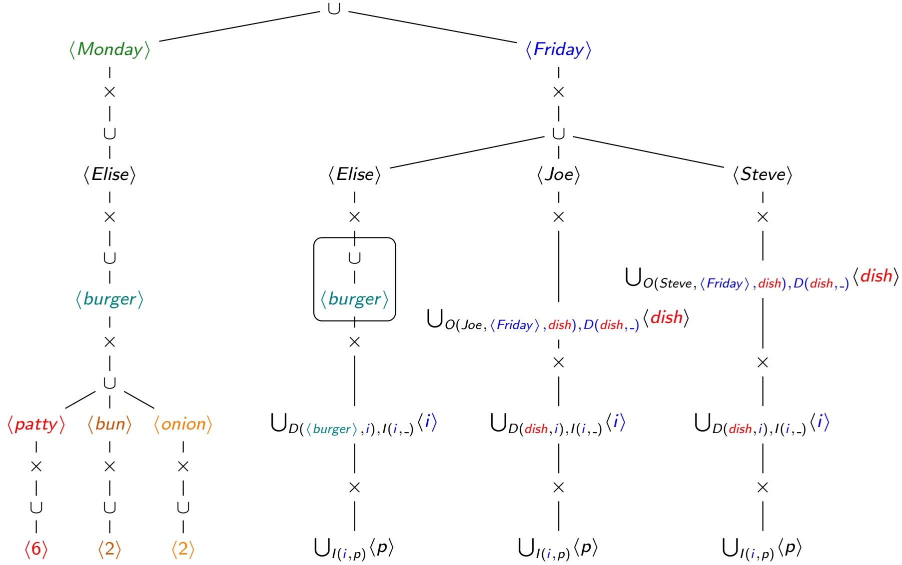
# Example: Computing the Full Join Result



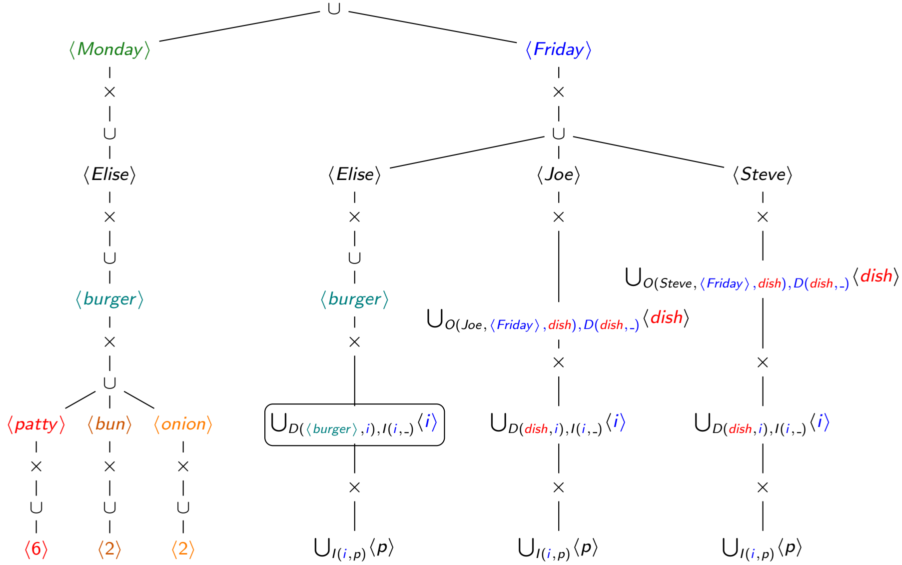
# Example: Computing the Full Join Result



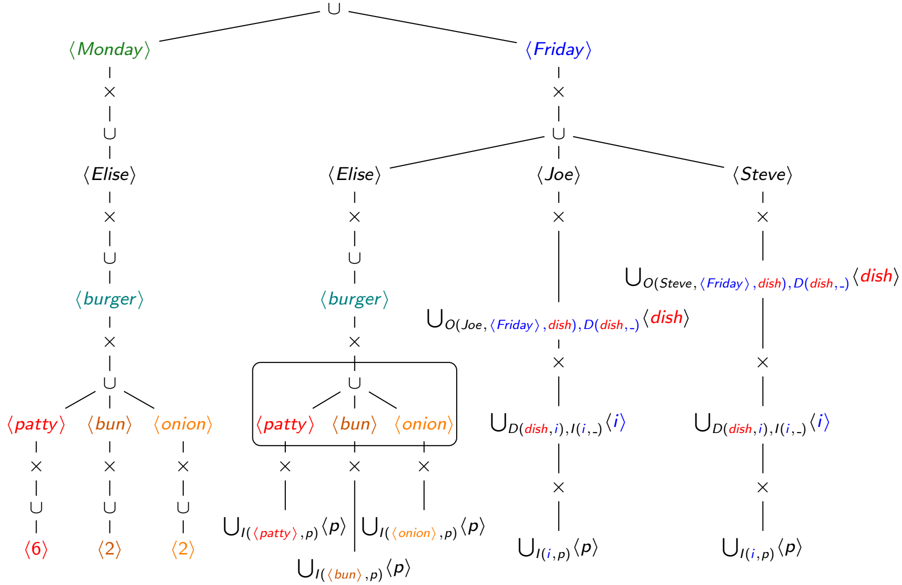
# Example: Computing the Full Join Result



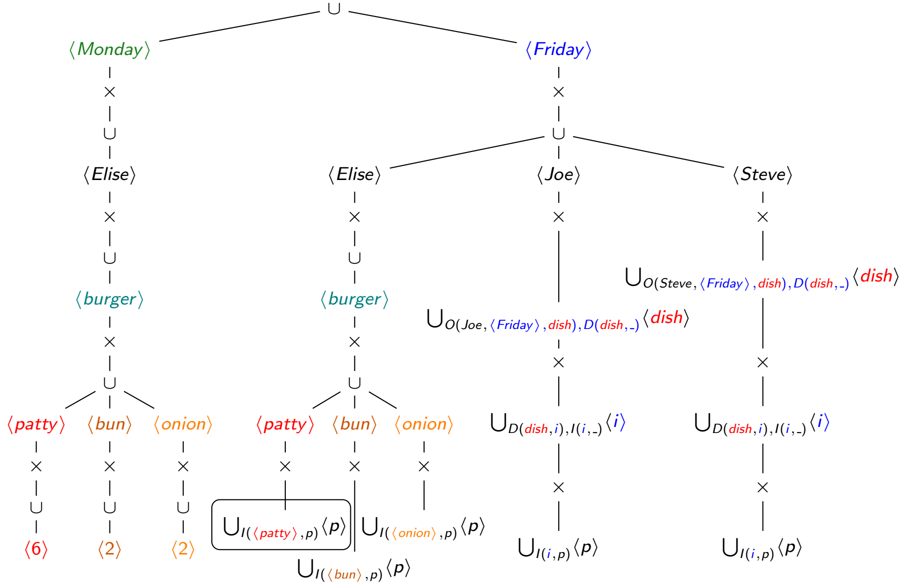
# Example: Computing the Full Join Result



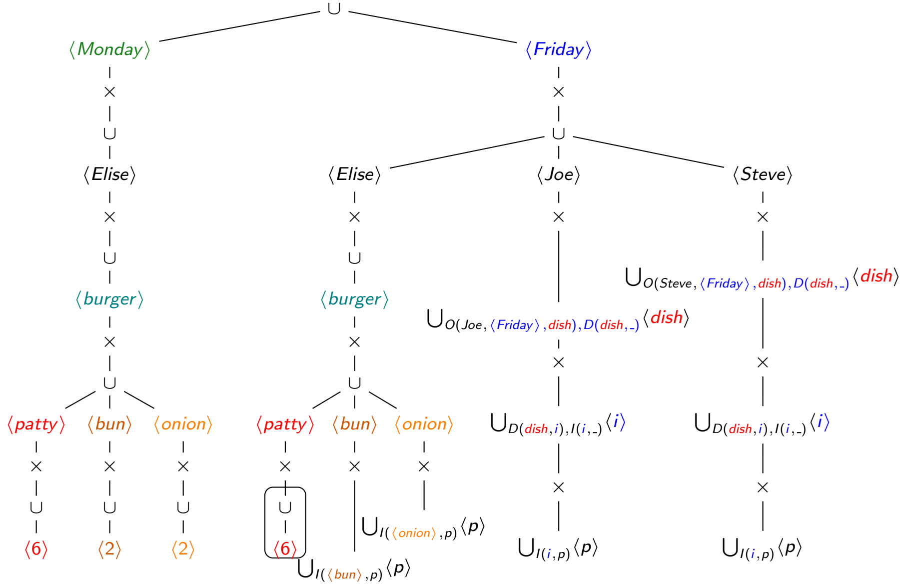
# Example: Computing the Full Join Result



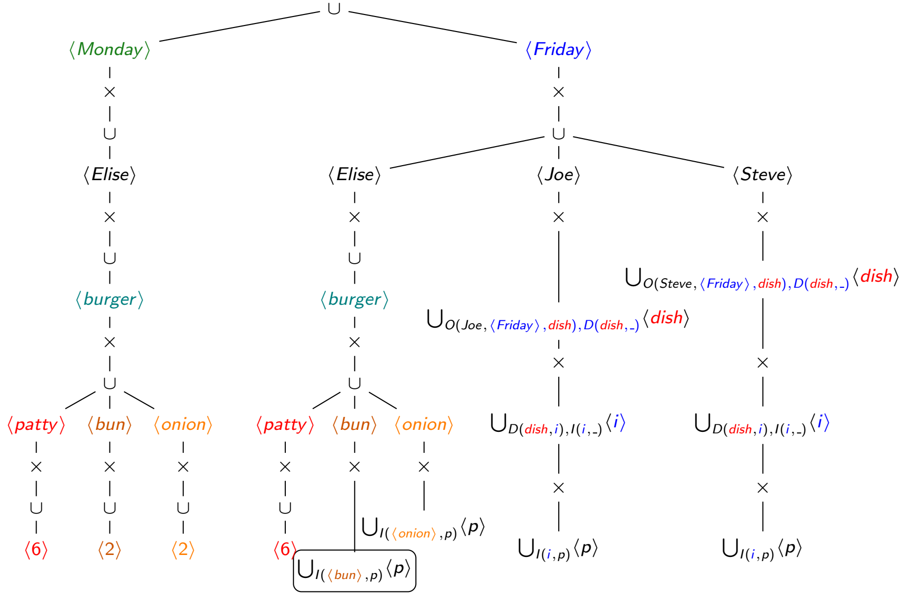
# Example: Computing the Full Join Result



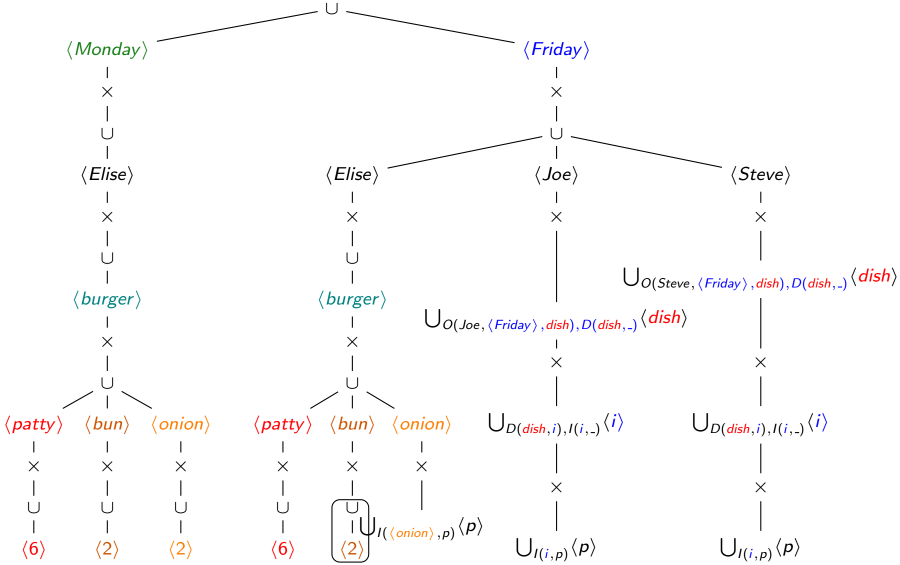
# Example: Computing the Full Join Result



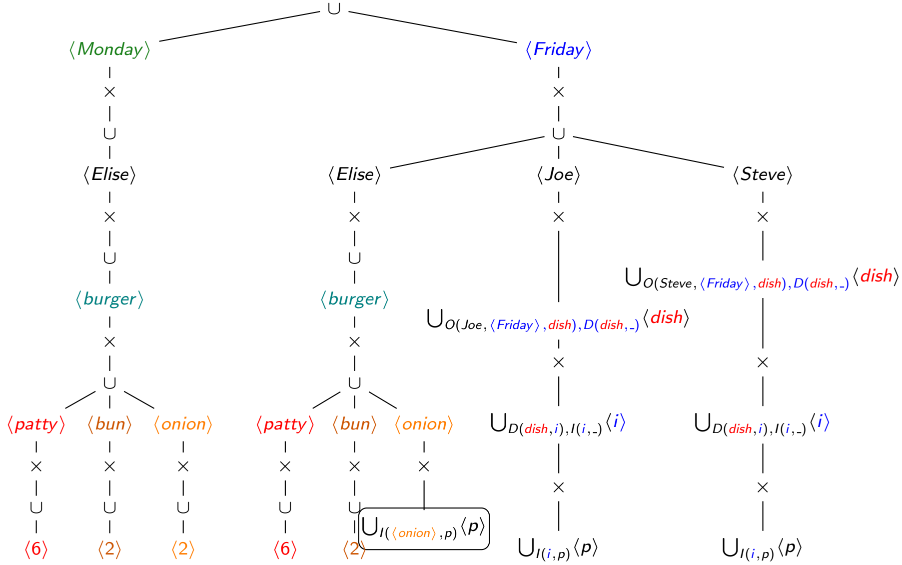
# Example: Computing the Full Join Result



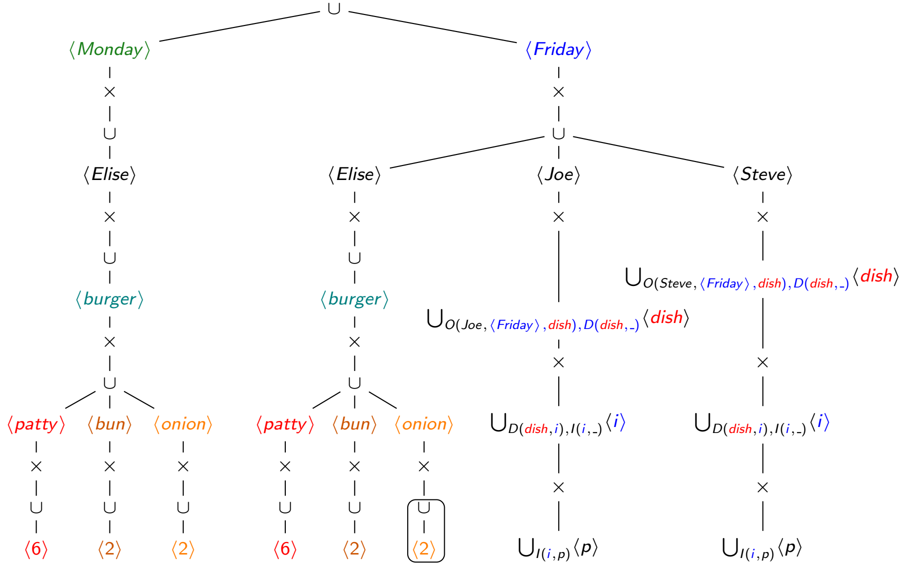
# Example: Computing the Full Join Result



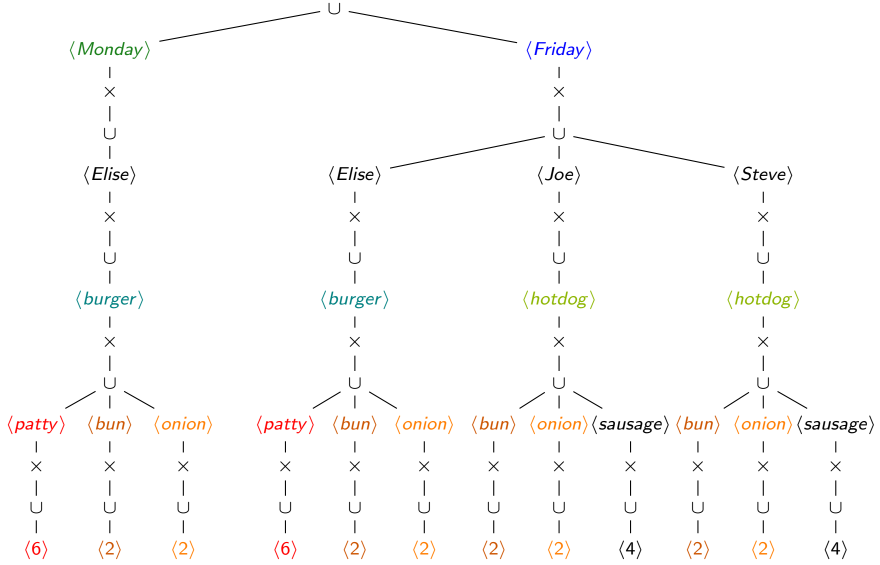
# Example: Computing the Full Join Result



# Example: Computing the Full Join Result

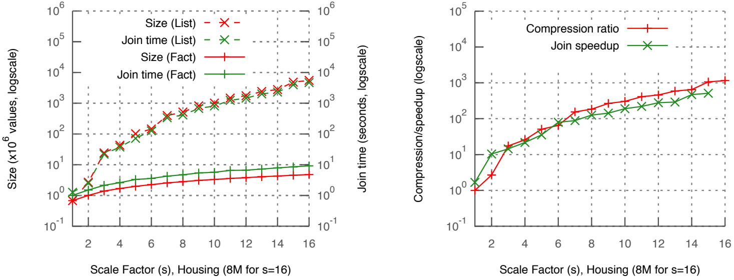


# Example: Computing the Full Join Result



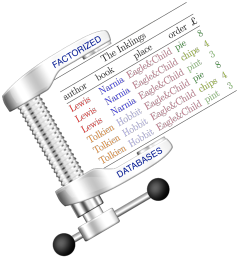
# Experiment: Factorized vs. Listing Computation

| | | Retailer (3B) | LastFM (5.8M) |
|---|---|---|---|
| Join Size (values) | Factorization | 169M | 316K |
| | Listing | 3.6B | 591M |
| | Compression | 21.4× | 1870.7× |
| Join Time (sec) | FDB | 30 | 10 |
| | PostgreSQL | 217 | 61 |
| | Speedup | 7× | 6.1× |




Both FDB and PostgreSQL list the records in the results of the join queries.

# Outline of Part 1: Joins



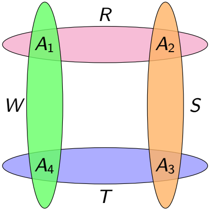
# Relevant Work not Covered in the Course

- Widths, results sizes, and join computation *under functional dependencies* [GLVV12,ANS16,GT17,ANS17]
- *Adaptive* join processing with lower complexity [AYZ97,ANS17]
  - We exemplify this next with the 4-cycle join [AYZ97]
- Covers: Relational counterpart of factorized representation [KO18]

# Recall the (4-cycle) Join

$$Q(A_1, A_2, A_3, A_4) = R(A_1, A_2), S(A_2, A_3), T(A_3, A_4), W(A_4, A_1).$$

The linear program for its fractional edge cover number:




$$\begin{array}{lccccccccc}
\multicolumn{10}{l}{\textit{minimize } x_R + x_S + x_T + x_W} \\
\multicolumn{10}{l}{\textit{subject to}} \\
A_1: & x_R & & & & & + & x_W & \geq 1 \\
A_2: & x_R & + & x_S & & & & & \geq 1 \\
A_3: & & & x_S & + & x_T & & & \geq 1 \\
A_4: & & & & & x_T & + & x_W & \geq 1 \\
& x_R \geq 0 & & x_S \geq 0 & & x_T \geq 0 & & x_W \geq 0 &
\end{array}$$

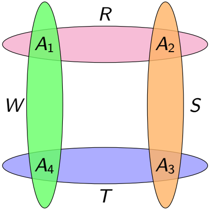
Solutions: $x_R = x_T = 1$ or $x_S = x_W = 1$. Then, $\rho^* = 2$. Also, fhtw $= 2$.

Lower bound $\Omega(N^2)$ obtained for
$R(A_1, A_2) = T(A_3, A_4) = [N] \times \{1\}$ and $S(A_2, A_3) = W(A_4, A_1) = \{1\} \times [N]$

- The variables $A_1$ and $A_3$ get values $[N]$
- The variable $A_2$ and $A_4$ get value $\{1\}$

# Can We Do The *Boolean* 4-Cycle Join Faster?

$$Q() = R(A_1, A_2), S(A_2, A_3), T(A_3, A_4), W(A_4, A_1).$$




We can use one of the two decompositions:

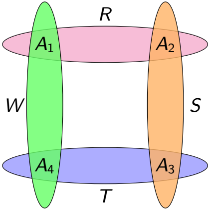
$$T_1 : \overbrace{\{A_1, A_2, A_3\}}^{B_1} - \overbrace{\{A_1, A_3, A_4\}}^{B_2}$$
$$T_2 : \underbrace{\{A_4, A_1, A_2\}}_{B_3} - \underbrace{\{A_2, A_3, A_4\}}_{B_4}$$

Lower-bound: $A_1$ and $A_3$ get values $[N]$ and $A_2$ and $A_4$ get value $\{1\}$.

- Use $T_1$: $\underbrace{R(A_1, A_2), S(A_2, A_3)}_{N \cdot N = N^2}$ cover $B_1$, $\underbrace{T(A_3, A_4), W(A_4, A_1)}_{N \cdot N = N^2}$ cover $B_2$

# Can We Do The *Boolean* 4-Cycle Join Faster?

$$Q() = R(A_1, A_2), S(A_2, A_3), T(A_3, A_4), W(A_4, A_1).$$




We can use one of the two decompositions:

$$T_1 : \overbrace{\{A_1, A_2, A_3\}}^{B_1} - \overbrace{\{A_1, A_3, A_4\}}^{B_2}$$

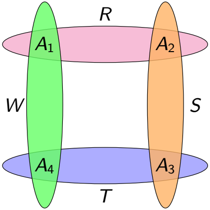
$$T_2 : \underbrace{\{A_4, A_1, A_2\}}_{B_3} - \underbrace{\{A_2, A_3, A_4\}}_{B_4}$$

Lower-bound: $A_1$ and $A_3$ get values $[N]$ and $A_2$ and $A_4$ get value $\{1\}$.

- Use $T_1$: $\underbrace{R(A_1, A_2), S(A_2, A_3)}_{N \cdot N = N^2}$ cover $B_1$, $\underbrace{T(A_3, A_4), W(A_4, A_1)}_{N \cdot N = N^2}$ cover $B_2$
- Use $T_2$: $\underbrace{R(A_1, A_2), W(A_4, A_1)}_{\mathbf{N}}$ cover $B_3$, $\underbrace{S(A_2, A_3), T(A_3, A_4)}_{\mathbf{N}}$ cover $B_4$

# Can We Do The *Boolean* 4-Cycle Join Faster?

$$Q() = R(A_1, A_2), S(A_2, A_3), T(A_3, A_4), W(A_4, A_1).$$




We can use one of the two decompositions:

$$T_1 : \overbrace{\{A_1, A_2, A_3\}}^{B_1} - \overbrace{\{A_1, A_3, A_4\}}^{B_2}$$
$$T_2 : \underbrace{\{A_4, A_1, A_2\}}_{B_3} - \underbrace{\{A_2, A_3, A_4\}}_{B_4}$$

Lower-bound: $A_1$ and $A_3$ get values $[N]$ and $A_2$ and $A_4$ get value $\{1\}$.

- Use $T_1$: $\underbrace{R(A_1, A_2), S(A_2, A_3)}_{N \cdot N = N^2}$ cover $B_1$, $\underbrace{T(A_3, A_4), W(A_4, A_1)}_{N \cdot N = N^2}$ cover $B_2$
- Use $T_2$: $\underbrace{R(A_1, A_2), W(A_4, A_1)}_{\mathbf{N}}$ cover $B_3$, $\underbrace{S(A_2, A_3), T(A_3, A_4)}_{\mathbf{N}}$ cover $B_4$

Idea: Why not use **different decompositions** for **different classes** of input databases or even for **different partitions** of a relation?

# Light and Heavy Values

Fix $\epsilon \in [0,1]$. A value $a$ of variable $A$ in relation $R$ is:

**HEAVY** if $|\sigma_{A=a}(R)| \geq N^\epsilon$ **LIGHT** if $|\sigma_{A=a}(R)| < N^\epsilon$

# Light and Heavy Values

Fix $\epsilon \in [0,1]$. A value $a$ of variable $A$ in relation $R$ is:

**HEAVY** if $|\sigma_{A=a}(R)| \geq N^{\epsilon}$ **LIGHT** if $|\sigma_{A=a}(R)| < N^{\epsilon}$

Partition $R(A_1, A_2)$ and $T(A_3, A_4)$ into heavy and light parts:

$$R = \underbrace{\{(a_1, a_2) \in R \mid a_1 \text{ is heavy}\}}_{R_h} \cup \underbrace{\{(a_1, a_2) \in R \mid a_1 \text{ is light}\}}_{R_l}$$

$$T = \underbrace{\{(a_3, a_4) \in T \mid a_3 \text{ is heavy}\}}_{T_h} \cup \underbrace{\{(a_3, a_4) \in T \mid a_3 \text{ is light}\}}_{T_l}$$

# Evaluation of the 4-Cycle Boolean Query in $O(N^{3/2})$

$$Q() = R(A_1, A_2), S(A_2, A_3), T(A_3, A_4), W(A_4, A_1)$$

Recall the two decompositions:

$$T_1 : \overbrace{\{A_1, A_2, A_3\}}^{B_1} - \overbrace{\{A_1, A_3, A_4\}}^{B_2} \quad T_2 : \overbrace{\{A_4, A_1, A_2\}}^{B_3} - \overbrace{\{A_2, A_3, A_4\}}^{B_4}$$

We rewrite $Q$ as $Q() = Q_1() \cup Q_2() \cup Q_3()$, where

$$Q_1() = \mathbf{R_h}(A_1, A_2), S(A_2, A_3), T(A_3, A_4), W(A_4, A_1)$$

$$Q_2() = \mathbf{R_l}(A_1, A_2), S(A_2, A_3), \mathbf{T_h}(A_3, A_4), W(A_4, A_1)$$

$$Q_3() = \mathbf{R_l}(A_1, A_2), S(A_2, A_3), \mathbf{T_l}(A_3, A_4), W(A_4, A_1)$$

# Evaluation of the 4-Cycle Boolean Query in $O(N^{3/2})$

$$Q() = R(A_1, A_2), S(A_2, A_3), T(A_3, A_4), W(A_4, A_1)$$

Recall the two decompositions:

$$T_1 : \overbrace{\{A_1, A_2, A_3\}}^{B_1} - \overbrace{\{A_1, A_3, A_4\}}^{B_2} \quad T_2 : \overbrace{\{A_4, A_1, A_2\}}^{B_3} - \overbrace{\{A_2, A_3, A_4\}}^{B_4}$$

We evaluate

$$Q_1() = \mathbf{R_h}(A_1, A_2), S(A_2, A_3), T(A_3, A_4), W(A_4, A_1)$$

using $T_1$: $\underbrace{\pi_{A_1} R_h(A_1), S(A_2, A_3)}_{N^{1-\epsilon} \cdot N = N^{2-\epsilon}}$ covers $B_1$, $\underbrace{\pi_{A_1} R_h(A_1), T(A_3, A_4)}_{N^{1-\epsilon} \cdot N = N^{2-\epsilon}}$ covers $B_2$

For $\epsilon = 1/2$, the time to compute $Q_1$ is $N^{3/2}$.

# Evaluation of the 4-Cycle Boolean Query in $O(N^{3/2})$

$$Q() = R(A_1, A_2), S(A_2, A_3), T(A_3, A_4), W(A_4, A_1)$$

Recall the two decompositions:

$$T_1 : \overbrace{\{A_1, A_2, A_3\}}^{B_1} - \overbrace{\{A_1, A_3, A_4\}}^{B_2} \quad T_2 : \overbrace{\{A_4, A_1, A_2\}}^{B_3} - \overbrace{\{A_2, A_3, A_4\}}^{B_4}$$

We evaluate

$$Q_2() = \mathbf{R_l}(A_1, A_2), S(A_2, A_3), \mathbf{T_h}(A_3, A_4), W(A_4, A_1)$$

using $T_1$: $\underbrace{\pi_{A_3} T_h(A_3), R_l(A_1, A_2)}_{N^{1-\epsilon} \cdot N = N^{2-\epsilon}}$ covers $B_1$, $\underbrace{\pi_{A_3} T_h(A_3), W(A_1, A_4)}_{N^{1-\epsilon} \cdot N = N^{2-\epsilon}}$ covers $B_2$

For $\epsilon = 1/2$, the time to compute $Q_2$ is $N^{3/2}$.

# Evaluation of the 4-Cycle Boolean Query in $O(N^{3/2})$

$$Q() = R(A_1, A_2), S(A_2, A_3), T(A_3, A_4), W(A_4, A_1)$$

Recall the two decompositions:

$$T_1 : \overbrace{\{A_1, A_2, A_3\}}^{B_1} - \overbrace{\{A_1, A_3, A_4\}}^{B_2} \quad T_2 : \overbrace{\{A_4, A_1, A_2\}}^{B_3} - \overbrace{\{A_2, A_3, A_4\}}^{B_4}$$

We evaluate

$$Q_3() = \mathbf{R_l}(A_1, A_2), S(A_2, A_3), \mathbf{T_l}(A_3, A_4), W(A_4, A_1)$$

using $T_2$: $\underbrace{W(A_4, A_1), R_l(A_1, A_2)}_{N \cdot N^{\epsilon} = N^{1+\epsilon}}$ covers $B_1$, $\underbrace{S(A_2, A_3), T_l(A_3, A_4)}_{N \cdot N^{\epsilon} = N^{1+\epsilon}}$ covers $B_2$

For $\epsilon = 1/2$, the time to compute $Q_3$ is $N^{3/2}$.

# Covers: Relational Counterparts of Factorizations

- Factorized representations are not relational :(
  - This makes it difficult to integrate them into relational data systems
- Covers of Query Results [KO17]
  - Relations that are lossless representations of query results, yet are as succinct as factorized representations
  - For a join query $Q$ and any database of size $N$, a cover has size $O(N^{fhtw(Q)})$ and can be computed in time $\widetilde{O}(N^{fhtw(Q)})$
- How to get a cover?
  - Construct a hypertree decomposition of the query
  - Project query result onto the bags of the hypertree decomposition
  - Construct on these projections the hypergraph of the query result
  - Take a minimal edge cover of this hypergraph

# Recall the Itemized Customer Orders Example

Orders (O for short)

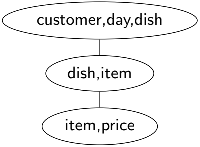
| customer | day | dish |
|---|---|---|
| Elise | Monday | burger |
| Elise | Friday | burger |
| Steve | Friday | hotdog |
| Joe | Friday | hotdog |

Dish (D for short)

| dish | item |
|---|---|
| burger | patty |
| burger | onion |
| burger | bun |
| hotdog | bun |
| hotdog | onion |
| hotdog | sausage |

Items (I for short)

| item | price |
|---|---|
| patty | 6 |
| onion | 2 |
| bun | 2 |
| sausage | 4 |



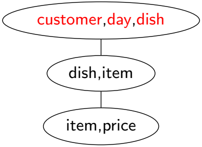

O(customer, day, dish), D(dish, item), I(item, price)

| customer | day | dish | item | price |
|---|---|---|---|---|
| Elise | Monday | burger | patty | 6 |
| Elise | Monday | burger | onion | 2 |
| Elise | Monday | burger | bun | 2 |
| Elise | Friday | burger | patty | 6 |
| Elise | Friday | burger | onion | 2 |
| Elise | Friday | burger | bun | 2 |
| ... | ... | ... | ... | ... |

# The Hypergraph of the Query Result

Elise Monday burger

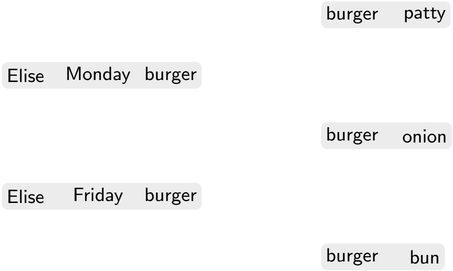
Elise Friday burger

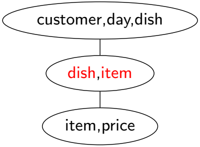



O(customer, day, dish), D(dish, item), I(item, price)

| customer | day | dish | item | price |
|---|---|---|---|---|
| Elise | Monday | burger | patty | 6 |
| Elise | Monday | burger | onion | 2 |
| Elise | Monday | burger | bun | 2 |
| Elise | Friday | burger | patty | 6 |
| Elise | Friday | burger | onion | 2 |
| Elise | Friday | burger | bun | 2 |
| ... | ... | ... | ... | ... |

# The Hypergraph of the Query Result




O(customer, day, dish), D(dish, item), I(item, price)

| customer | day | dish | item | price |
|---|---|---|---|---|
| Elise | Monday | burger | patty | 6 |
| Elise | Monday | burger | onion | 2 |
| Elise | Monday | burger | bun | 2 |
| Elise | Friday | burger | patty | 6 |
| Elise | Friday | burger | onion | 2 |
| Elise | Friday | burger | bun | 2 |
| ... | ... | ... | ... | ... |

# The Hypergraph of the Query Result

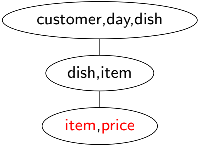
burger patty — patty 6

Elise Monday burger

burger onion — onion 2

Elise Friday burger

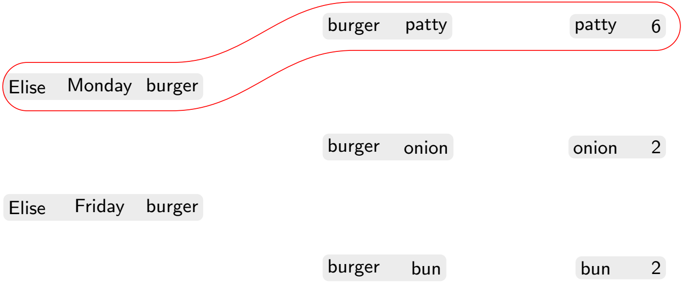
burger bun — bun 2

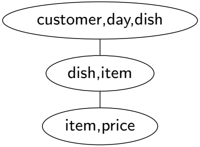



O(customer, day, dish), D(dish, item), I(item, price)

| customer | day | dish | item | price |
|---|---|---|---|---|
| Elise | Monday | burger | patty | 6 |
| Elise | Monday | burger | onion | 2 |
| Elise | Monday | burger | bun | 2 |
| Elise | Friday | burger | patty | 6 |
| Elise | Friday | burger | onion | 2 |
| Elise | Friday | burger | bun | 2 |
| ... | ... | ... | ... | ... |

# The Hypergraph of the Query Result

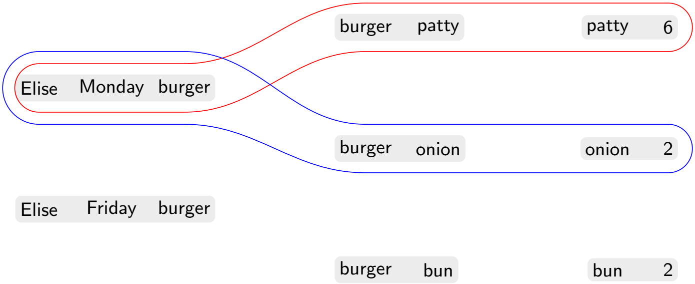


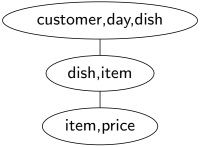




O(customer, day, dish), D(dish, item), I(item, price)

| customer | day | dish | item | price |
|---|---|---|---|---|
| Elise | Monday | burger | patty | 6 |
| Elise | Monday | burger | onion | 2 |
| Elise | Monday | burger | bun | 2 |
| Elise | Friday | burger | patty | 6 |
| Elise | Friday | burger | onion | 2 |
| Elise | Friday | burger | bun | 2 |
| ... | ... | ... | ... | ... |

# The Hypergraph of the Query Result

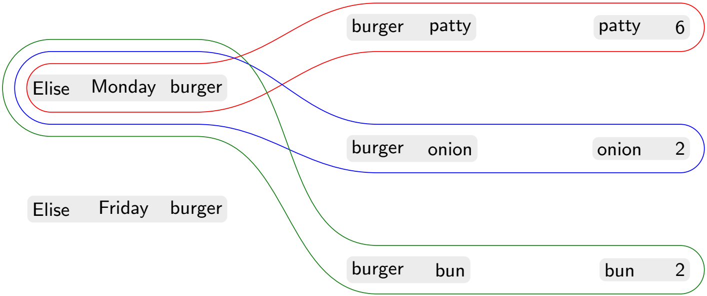


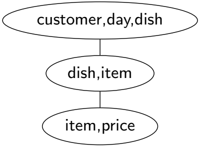




O(customer, day, dish), D(dish, item), I(item, price)

| customer | day | dish | item | price |
|---|---|---|---|---|
| Elise | Monday | burger | patty | 6 |
| Elise | Monday | burger | onion | 2 |
| Elise | Monday | burger | bun | 2 |
| Elise | Friday | burger | patty | 6 |
| Elise | Friday | burger | onion | 2 |
| Elise | Friday | burger | bun | 2 |
| ... | ... | ... | ... | ... |

# The Hypergraph of the Query Result

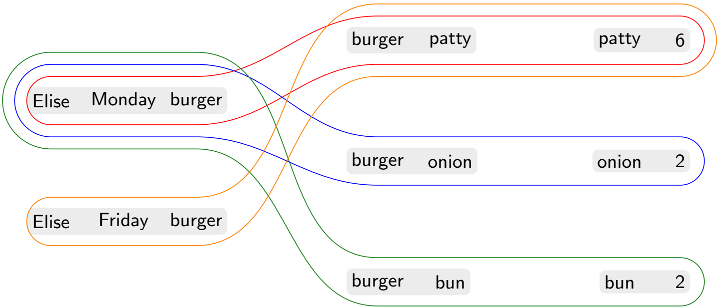


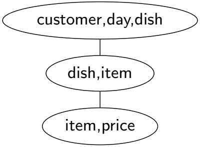




O(customer, day, dish), D(dish, item), I(item, price)

| customer | day | dish | item | price |
|---|---|---|---|---|
| Elise | Monday | burger | patty | 6 |
| Elise | Monday | burger | onion | 2 |
| Elise | Monday | burger | bun | 2 |
| Elise | Friday | burger | patty | 6 |
| Elise | Friday | burger | onion | 2 |
| Elise | Friday | burger | bun | 2 |
| ... | ... | ... | ... | ... |

# The Hypergraph of the Query Result

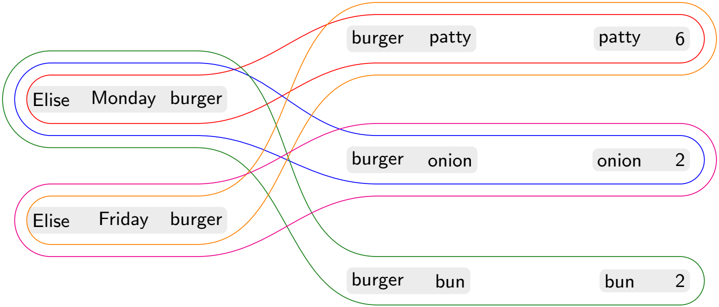


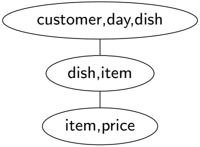




O(customer, day, dish), D(dish, item), I(item, price)

| customer | day | dish | item | price |
|---|---|---|---|---|
| Elise | Monday | burger | patty | 6 |
| Elise | Monday | burger | onion | 2 |
| Elise | Monday | burger | bun | 2 |
| Elise | Friday | burger | patty | 6 |
| Elise | Friday | burger | onion | 2 |
| Elise | Friday | burger | bun | 2 |
| ... | ... | ... | ... | ... |

# The Hypergraph of the Query Result

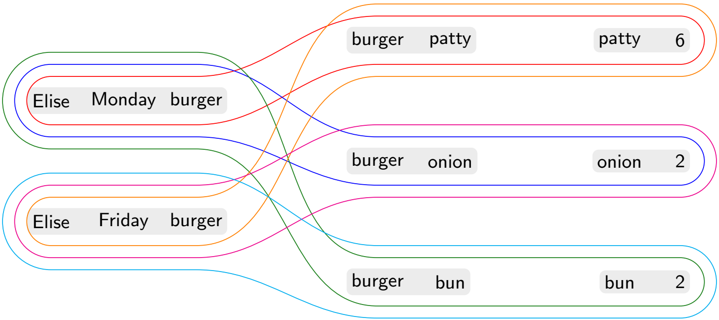


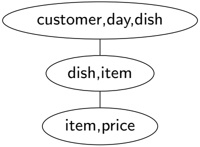




O(customer, day, dish), D(dish, item), I(item, price)

| customer | day | dish | item | price |
|---|---|---|---|---|
| Elise | Monday | burger | patty | 6 |
| Elise | Monday | burger | onion | 2 |
| Elise | Monday | burger | bun | 2 |
| Elise | Friday | burger | patty | 6 |
| Elise | Friday | burger | onion | 2 |
| Elise | Friday | burger | bun | 2 |
| ... | ... | ... | ... | ... |

# The Hypergraph of the Query Result

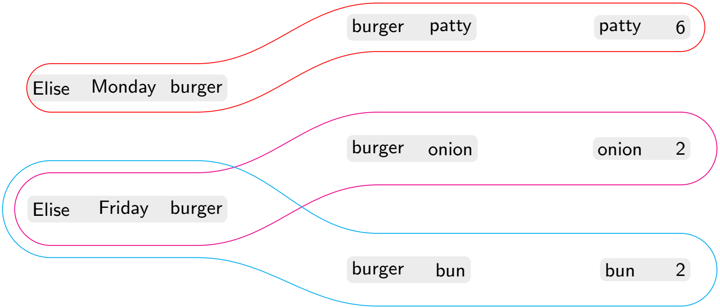


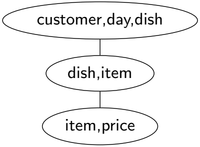




O(customer, day, dish), D(dish, item), I(item, price)

| customer | day | dish | item | price |
|---|---|---|---|---|
| Elise | Monday | burger | patty | 6 |
| Elise | Monday | burger | onion | 2 |
| Elise | Monday | burger | bun | 2 |
| Elise | Friday | burger | patty | 6 |
| Elise | Friday | burger | onion | 2 |
| Elise | Friday | burger | bun | 2 |
| ... | ... | ... | ... | ... |

# A Minimal Edge Cover of the Hypergraph



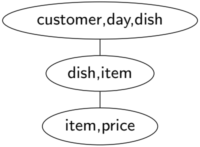

O(customer, day, dish), D(dish, item), I(item, price)

| customer | day | dish | item | price |
|---|---|---|---|---|
| Elise | Monday | burger | patty | 6 |
| Elise | Monday | burger | onion | 2 |
| Elise | Monday | burger | bun | 2 |
| Elise | Friday | burger | patty | 6 |
| Elise | Friday | burger | onion | 2 |
| Elise | Friday | burger | bun | 2 |
| ... | ... | ... | ... | ... |

# A Cover of (a part of) the Query Result

O(customer, day, dish), D(dish, item), I(item, price)

| customer | day | dish | item | price |
|---|---|---|---|---|
| Elise | Monday | burger | patty | 6 |
| Elise | Friday | burger | onion | 2 |
| Elise | Friday | burger | bun | 2 |




O(customer, day, dish), D(dish, item), I(item, price)

| customer | day | dish | item | price |
|---|---|---|---|---|
| Elise | Monday | burger | patty | 6 |
| Elise | Monday | burger | onion | 2 |
| Elise | Monday | burger | bun | 2 |
| Elise | Friday | burger | patty | 6 |
| Elise | Friday | burger | onion | 2 |
| Elise | Friday | burger | bun | 2 |
| ... | ... | ... | ... | ... |

# References


LW49 **An inequality related to the isoperimetric inequality.**
Loomis, Whitney. In Bull. Amer. Math. Soc., 55 (1949).
https://www.ams.org/journals/bull/1949-55-10/

A81 **On the number of subgraphs of prescribed type of graphs with a given number of edges.**
Alon. In Israel J. Math., 38 (1981).
https://link.springer.com/content/pdf/10.1007/BF02761855.pdf

BT95 **Projections of bodies and hereditary properties of hypergraphs.**
Bollobaás, Thomason. In Bull. London Math. Soc., 27 (1995).
https://pdfs.semanticscholar.org/02c2/9f48e698ccbe7854be8012439c535453634f.pdf

AYZ97 **Finding and counting given length cycles.**
Alon, Yuster, Zwick. In Algorithmica 17, 3 (1997).
https://m.tau.ac.il/~nogaa/PDFS/ayz97.pdf

GLS99 **Hypertree decompositions and tractable queries.**
Gottlob, Leone, Scarcello. In PODS 1999.
https://arxiv.org/abs/cs/9812022

AGM08 **Size bounds and query plans for relational joins.**
Atserias, Grohe, Marx. In FOCS 2008 and SIAM J. Comput., 42(4) 2013.
http://epubs.siam.org/doi/10.1137/110859440

# References


M10 **Approximating fractional hypertree width.**
Marx. In ACM TALG 2010.
http://dl.acm.org/citation.cfm?id=1721845

NPRR12 **Worst-case optimal join algorithms: [extended abstract]**
Ngo, Porat, Ré, Rudra. In PODS 2012.
http://dl.acm.org/citation.cfm?id=2213565

OZ12 **Factorised representations of query results: size bounds and readability.**
Olteanu, Zavodny. In ICDT 2012.
http://dl.acm.org/citation.cfm?doid=2274576.2274607
Also https://arxiv.org/abs/1104.0867, April 2011.

GLVV12 **Size and treewidth bounds for conjunctive queries.**
Gottlob, Lee, Valiant, Valiant. In J. ACM, 59 (2012).
https://www.cs.ox.ac.uk/files/5024/GLVV_7_11_conjqueries_jacm.pdf

NRR13 **Skew Strikes Back: New Developments in the Theory of Join Algorithms.**
Ngo, Ré, Rudra. In SIGMOD Rec. 2013.
https://arxiv.org/abs/1310.3314

V14 **Triejoin: A Simple, Worst-Case Optimal Join Algorithm.**
Veldhuizen. In ICDT 2014.
http://openproceedings.org/ICDT/2014/paper_13.pdf

# References


OZ15 **Size Bounds for Factorised Representations of Query Results.**
Olteanu, Zavodny. In ACM TODS 2015 (submitted July 2013).
http://dl.acm.org/citation.cfm?doid=2656335

ANS16 **Computing join queries with functional dependencies.**
Abo Khamis, Ngo, Suciu. In PODS 2017.
https://arxiv.org/abs/1604.00111

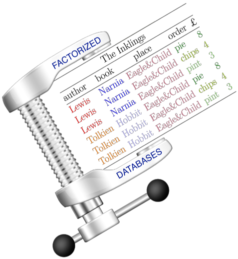
GT17 **Entropy Bounds for Conjunctive Queries with Functional Dependencies.**
Gogacz, Torunczyk. In ICDT 2017.
http://drops.dagstuhl.de/opus/volltexte/2017/7047/

ANS17 **What do Shannon-type inequalities, submodular width, and disjunctive Datalog have to do with one another?**
Abo Khamis, Ngo, Suciu. In PODS 2017.
https://arxiv.org/abs/1612.02503

KO18 **Covers of Query Results.**
Kara, Olteanu. In ICDT 2018.
https://arxiv.org/abs/1709.01600

N18 **Worst-Case Optimal Join Algorithms: Techniques, Results, and Open Problems.**
Ngo. In PODS 2018.
https://arxiv.org/abs/1803.09930

# Outline of Part 1: Joins




Introduction by Examples

Decompositions and Variable Orders

Size Bounds for Join Results

Worst-Case Optimal Join Algorithms

Further Work and References

Quiz

# QUIZ on Joins (1/4)

For each of the following queries, please show the following:

1. A hypertree decomposition and an equivalent variable order
2. The fractional edge cover number and the fractional hypertree width

Path Query of length $n$:

$$P_n(X_1, \ldots, X_{n+1}) = R_1(X_1, X_2), R_2(X_2, X_3), R_3(X_3, X_4), \ldots, R_n(X_n, X_{n+1}).$$

# QUIZ on Joins (2/4)

For each of the following queries, please show the following:

1. A hypertree decomposition and an equivalent variable order
2. The fractional edge cover number and the fractional hypertree width

Loop Query of length $n$:

$$L_n(X_1, \ldots, X_{n+1}) = R_1(X_1, X_2), R_2(X_2, X_3), R_3(X_3, X_4), \ldots, R_n(X_n, X_1).$$

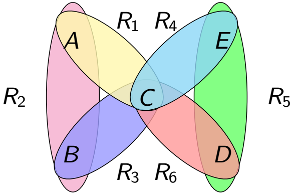
# QUIZ on Joins (3/4)

For each of the following queries, please show the following:

1. A hypertree decomposition and an equivalent variable order
2. The fractional edge cover number and the fractional hypertree width

Bowtie Query:

$$Q_{\bowtie}(A, B, C, D, E) = R_1(A, C), R_2(A, B), R_3(B, C), R_4(C, E), R_5(E, D), R_6(C, D).$$



# QUIZ on Joins (4/4)

For each of the following queries, please show the following:

1. A hypertree decomposition and an equivalent variable order
2. The fractional edge cover number and the fractional hypertree width

Loomis-Whitney Queries of length $n$: A $LW_n$ query has $n$ variables $X_1, \ldots, X_n$ and $n$ relation symbols such that for every $i \in [n]$ the relation symbol $R_i$ has variables $\{X_1, \ldots, X_n\} - \{X_i\}$:

$$LW_n(X_1, \ldots, X_n) = R_1(X_2, \ldots, X_n), \ldots, R_i(X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_n), \ldots, \\ R_n(X_1, \ldots, X_{n-1})$$

$LW_n$ captures the Loomis–Whitney inequality: Estimate the "size" of a $d$-dimensional set by the sizes of its $(d-1)$-dimensional projections.

$LW_3$ is the triangle query.